Defeating Deferral: A Proposal for Retrospective Taxation

This paper begins where an earlier paper, *Contingent Payments and the Time Value of Money*,[1] left off. That paper concluded with an appeal for a "yield-based" approach to the taxation of contingent payments, but did not fully explore the ramifications of the yield-based approach.

The issue never left my mind in the decade that separates that paper from this one. I had the feeling, and still do, that the growth of derivatives poses a serious challenge to the tax law's attempt to tax different classes of income differently. Also, I have always viewed the combination of deferral and the capital gains preference as compounding the most serious problems of horizontal and vertical inequity in our tax system. While others debate how large the capital tax preference should be, this paper offers reasons why the tax on capital gains should be *higher* than that on ordinary income. The yield-based approach quantifies just how much higher that tax should be.

I found it astonishing that after nearly a century of the income tax, there was still something original to say about the realization requirement. My goal was measure the benefits of tax deferral, so that appropriate adjustments could be made that would precisely offset those benefits. But a disturbing fact emerged: the resulting tax was *non-linear*, in the sense that, even with a fixed rate of tax, the taxes imposed on two separate investments was greater than the tax imposed on those same investments viewed together. This characteristic would seem to contravene any minimum standard of rationality that any tax system should satisfy.

1 Land, *infra* note 5.

Yet rather than being a fatal flaw, this feature of the yield-based approach turned out to be a fertile source of insights into how taxpayers make use of tax deferral, and how any system that corrects for it must look beyond individual assets to the entire scope of a taxpayer's activities, including not just an asset portfolio but also liabilities and instruments, such as swaps, that are neither assets nor liabilities.

There is math in this paper, but I have tried to keep it at the high school level. I have relegated to the Appendix a bit of calculus that is needed to prove one particular point. Otherwise, the mathematical demands are limited to a willingness to deal with exponents and logarithms. Regrettably, these are unavoidable when dealing with a topic that is bound up with growth and compounding of income.

Contents

I. INTRODUCTION

The next best thing to not paying tax is putting it off. This principle, more than any other, has motivated tax planning over the years. In the quest for tax deferral, a taxpayer's best friend is the realization requirement: gain from an appreciated asset is typically not taxed until a sale or other realization event. Too much deferral, however, threatens the federal revenue. Indeed, much of the complexity in the tax law comes from provisions designed to restrict deferral.

How much is tax deferral worth? Deferral is sometimes described as an interest-free loan from the government. Is it worth the taxpayer's borrowing cost? The taxpayer's overall cost of capital, including equity? The government's borrowing cost? At short-term or long-term rates? All of these possibilities have been bruited about by commentators.[2]

This article proposes a different way of looking at tax deferral. Under this view, the tax deferred represents less of a loan from the government than an equity investment by the government in the appreciated asset.[3] Such an investment should participate on a pro rata basis in any future earnings from the asset. Not having to pay this share of earnings to the government is the value of deferring the tax.

Two surprising propositions flow from this view of tax deferral. First: *The value of tax deferral is independent of the length of the holding period.* If you double your money in two years or twenty, the value of being able to defer tax until sale is the same. Second: *The value of tax deferral is independent of whether the appreciation occurred early or late in the holding period.* This second proposition can be made even stronger, because the value of tax deferral is the same even if the asset value fluctuates wildly over the holding period, and in portions of the hold-

2 *See infra* Part II.B.3 (p. 303).

3 *See* Mary Louise Fellows, *A Comprehensive Attack on Tax Deferral*, 88 MICH. L. REV. 722, 749 n.63 (1990).

ing period the value goes down as well as up. The value of deferral depends on how much is earned, not on when it is earned. These conclusions are sufficiently counter-intuitive that some form of formal proofs are called for. These I provide,[4] but how to measure the value of deferral is a question of tax policy as much as mathematics.

The propositions are of more than academic interest. They point the way to eliminate the tax benefits of deferral. The proposed solution is a form of "retrospective taxation," which preserves the realization requirement but adjusts the tax to offset the benefit of deferral. The tax is adjusted so that the after-tax sale proceeds provide a yield on the investment that is equal to its pre-tax yield reduced by the tax rate.[5] Such a tax is immune to "strategic trading": that is, a taxpayer cannot alter the tax burden by trading in and out of the investment. Moreover, the tax as computed in this fashion is the *only* form of retrospective taxation of gains and losses that has this property.

Part I of this article looks briefly at the realization requirement and prior efforts to deal with its faults. Part III sets out the proposed method of retrospective taxation in a manner that taxes each investment separately. Part I shows how flaws in the separate-investment approach can be cured by relating retrospective taxation to the performance of the investment portfolio as a whole. Finally, Part V

4 *See* Part III.B (p. 317).

5 This particular method of retrospective taxation has been considered before, by me and others. *See, e.g.,* Cynthia J. Blum, *New Role for Treasury: Charging Interest on Tax Deferral Loans,* 25 HARV. J. LEGIS. 1, 14–15 (1988); Fellows, *supra* note 3, at 748–51; Mark P. Gergen, *The Effects of Price Volatility and Strategic Trading Under Realization, Expected Return and Retrospective Taxation,* 49 TAX L. REV. 209, 234–35 (1994); Stephen B. Land, *Contingent Payments and the Time Value of Money,* 40 TAX LAW. 237, 283–88 (1987) *reprinted in* STEPHEN B. LAND, I PAPERS ON TAXATION 47, 128–32 (2013); Alvin C. Warren, *Financial Contract Innovation and Income Policy,* 107 Harv. L. Rev. 460, 477–478 (1993). What does not appear to have been noticed before is that this method is free of the flaws commonly attributed to retrospective taxation based on charging interest on the tax deferred. *See infra* Part III.C (p. 320).

speculates about what life under the income tax would be like if this form of retrospective taxation were to be adopted.

II. THE REALIZATION REQUIREMENT

A. *The Dilemma*

Economically, income is earned as an asset appreciates, not when it is sold. There is a wide consensus that a theoretically ideal income tax would be imposed on the "Haig-Simons" conception of income, which is the value of the taxpayer's consumption during the relevant accounting period plus any increase (or minus any decrease) in the fair market value of the taxpayer's wealth.[6] Such a tax would have no realization requirement: asset appreciation is income, whether the asset is sold or not. Viewed against this standard, the realization requirement mismeasures income every time it applies. Yet the realization requirement is widely considered to be essential to make the tax system administrable. What can be done?

1. *Why We Can't Live With It*

A host of evils can be attributed to the realization requirement. It is unfair and inefficient; it makes investing riskier; and it increases

[6] HENRY C. SIMONS, PERSONAL INCOME TAXATION 41–58 (1938); Robert M. Haig, *The Concept of Income—Economic and Legal Aspects,* in THE FEDERAL INCOME TAX 1, 7 (Robert M. Haig, ed. 1921), *reprinted in* AM. ECON. ASS'N, READINGS IN THE ECONOMICS OF TAXATION 54, 59 (Richard A. Musgrave and Carl S. Shoup, eds. 1959). Although the Haig-Simons norm is a widely accepted ideal for an *income* tax, some have argued that it would be better to replace the income tax with a tax on consumption, allowing wealth to accumulate tax-free. *See, e.g.,* William D. Andrews, *A Consumption-Type or Cash Flow Personal Income Tax,* 87 HARV. L. REV. 1113 (1974). This article steers clear of that debate, but is nonetheless relevant to it. Much of the motivation for a consumption tax is to avoid the problems associated with the realization requirement. *See* William D. Andrews, *The Achilles' Heel of the Comprehensive Income Tax,* in NEW DIRECTIONS IN FEDERAL TAX POLICY FOR THE 1980'S, at 278, 280 (1986). To the extent that retrospective taxation can alleviate these problems, the case for a consumption tax becomes less compelling. Moreover, retrospective taxation can be implemented in a manner that achieves an income tax result on a consumption tax base. *See infra* Part IV.C.4 (p. 355).

the complexity of tax law. These problems have been extensively catalogued by others;[7] the most prominent problems are summarized here.

a. Horizontal Inequity. Horizontal equity is the principle that taxpayers with equal incomes should pay equal amounts of income tax. Suppose Andrea and Bob each own shares of GM stock that have gone up $30,000 in value; Andrea sells her shares and buys Ford stock instead. Andrea pays tax on her gain, but Bob does not, even though both have made the same amount of money on their investments.

Moreover, the realization requirement favors income from capital over income from labor. Charlotte, who has no investments but earns $30,000 in salary, lacks the choices available to Andrea and Bob; she must pay tax on her income, come what may.

b. Vertical Inequity. Vertical equity is the principle that taxpayers with more income should pay more tax than those with less. Under progressive taxation, the effective rate of tax on the rich should be higher than on the poor.

The realization requirement undermines progressive taxation. Bill Gates, arguably the richest man in the world,[8] has paid no tax on the appreciation in his Microsoft stock. Recent trends in the tax law have been kind to the rich. The effective rate of tax borne by the top 1% of taxpayers dropped from 35.5% in 1977 to 28.8% in 1993,[9] notwithstanding recent increases in top marginal rate.[10] The realization

7 *E.g.,* Deborah H. Schenk, *Taxation of Equity Derivatives: A Partial Integration Proposal*, 50 TAX L. REV. 571, 631–35 (1995); David J. Shakow, *Taxation Without Realization: A Proposal for Accrual Taxation*, 134 U. PA. L. REV. 1111, 1114–18 (1986).

8 His net worth was recently reported to be $23.9 billion. *In Search of the Real Bill Gates*, TIME, January 13, 1997, at 44.

9 Martin J. McMahon, Jr., *Renewing Progressive Taxation*, 60 TAX NOTES 109, 110 (July 5, 1993).

10 The Omnibus Budget Reconciliation Act of 1990, Pub. L. No. 101-508, § 11101(a), 104 Stat. 1388, 1388–403 (1992) raised the top marginal rate from 28% to 31%. The Omnibus Budget Reconciliation Act of 1993, Pub. L. No. 103-66, § 13202(a)(1), 107 Stat. 312, 461 (1993) raised the top rate to 39.6%. The maximum rate on long-term capital gains, however, recently fell to 20% for assets held for more than 18 months. The recent amendment also provides for an

requirement exacerbates this trend, because the rich derive a disproportionate part of their income from capital.

c. Lock-In Effect. The realization requirement creates the "lock-in" effect of current law, which discourages taxpayers, from selling assets in circumstances where, absent tax considerations, it would make sense to do so. Dave's stake in the business he founded is worth $5 million. He is ready to retire, and would like to sell out and diversify his investments. If he does so, however, he will have about one-third less to live on than if he holds on to his stake. Whatever his decision, tax planning will be a major consideration.[11]

The lock-in effect itself has been cited as a justification for a lower rate of tax on capital gains.[12] While a lower rate undoubtedly reduces the lock-in effect, it compounds the horizontal and vertical equity problems: capital gains enjoy the double benefit of deferral and a lower rate.

d. Loss Limitations. Although the realization requirement applies to losses as well as gains, the failure to derive any tax benefit from unrealized losses is mitigated by the fact that investors can choose to sell at any time, claim the tax loss, and reinvest in something similar. While the wash sale rules disallow tax losses from sales of stock or securities where the proceeds are reinvested in substantially identical stock or securities,[13] they do not preclude reinvestment in other stocks or securities that offer a similar profile of risk and reward. Because investors can choose when to sell, there is nothing symmetric about applying the realization requirement to both gains

18% maximum rate for assets acquired after December 31, 2000 and held for more than five years. I.R.C. § 1(h), amended by the Taxpayer Relief Act of 1997, Pub. L. No. 105-34, § 311(a), 111 Stat. 788, 831–32.

11 *See* Nancy L. Jacob, *Taxes, Investment Strategy and Diversifying Low-Basis Stock,* TRUSTS & ESTATES, May 1995, at 8.

12 Walter J. Blum, *A Handy Summary of the Capital Gains Arguments*, 35 TAXES 247, 256–59 (1957); Noël B. Cunningham & Deborah H. Schenk, *The Case for a Capital Gains Preference*, 48 TAX L. REV. 319, 344–74 (1993).

13 I.R.C. § 1091(a).

and losses. For this reason, the tax law limits the deductibility of realized losses that exceed realized gains.[14]

Even with these limitations, taxpayers can develop trading strategies that magnify the benefits of capital gains deferral. A tax-wise portfolio investment manager selectively sells loss investments to shelter any tax on investments that are sold at a gain. This strategy enables the portfolio as a whole to enjoy greater deferral benefits than any particular investment within it.[15]

Although the loss limitations lack teeth in most circumstances, they impose a serious burden on unfortunate investors that suffer losses on their portfolios as a whole. These investors have suffered a real economic loss, but are not entitled to apply any resulting tax losses against ordinary income.[16] As a result, they may end up paying substantial amounts of tax on their ordinary income, even though, taking the capital losses into account, they may have only broken even or worse.

Forcing taxpayers with zero or negative net income to pay an income tax is not merely offensive to notions of horizontal and vertical equity. It makes investing riskier. The tax law promises deferral and a favorable rate to investment winners; but denies a tax benefit to the losers. This magnification of risk should be a serious concern to those who want the tax law to promote investment.[17]

[14] Individuals can deduct only $3,000 of net capital losses annually, but can carry forward the excess as a loss deduction in future years, subject to the same restriction. Corporations cannot deduct any net capital losses, but can carry these losses back for up to three years and forward for up to five years. I.R.C. §§ 1211, 1212.

[15] *See* Gergen, *supra* note 5, at 212; Jeff Strnad, *The Taxation of Bonds: The Tax Trading Dimension,* 81 VA. L. REV. 47, 52–54, 75–79 (1995); Jeff Strnad, *Periodicity and Accretion Taxation: Norms and Implementation,* 99 YALE L.J. 1817, 1874–84 (1990).

[16] *See supra* note 14.

[17] The overall effect of the realization requirement and loss limitations on investment risk is hard to assess, because they affect different types of risk differently. Exposure to risks that are specific to particular investments in a larger portfolio is rewarded, because of the ability to realize losses selectively to offset gains; but

There would be no need for these loss limitations if there were no realization requirement.[18] Taxpayers would be required to report both unrealized gains and losses; actual trading would be irrelevant. There would be no opportunity for taxpayers to realize losses selectively while deferring the realization of gains, so special limitations on loss deductibility would be unnecessary.

e. Magnified Deferral. Tax planning can magnify the benefits of deferral. While some strategies involve exotic new financial instruments, one common approach is of venerable antiquity: the short sale against the box. Elaine would like to sell a stock investment with a $100,000 gain, but defer the tax on the gain to next year. She can eliminate her investment exposure immediately without triggering any tax liability by selling short an equal number of shares of the same stock. The following year, she can close out the short sale by delivering the shares she owns, and report her $100,000 gain in that year. The constructive sale rules added by the 1997 budget legislation foreclosed this strategy, but these rules continue to allow deferral even after much of the risk of ownership has been hedged away.[19]

The benefits of deferral can be even greater when tax rates are expected to drop. Not only is the tax deferred, the gain is taxed at a

exposure to general market risk is penalized, because of the limitations on overall losses. *See generally* Robert H. Scarborough, *Risk, Diversification and the Design of Loss Limitations Under a Realization-Based Income Tax,* 48 TAX L. REV. 677 (1993).

18 Additional restrictions on the deductibility of losses apply to taxpayers that have negative net income, even after ordinary income items are taken into account. I.R.C. § 172. The tax law allows limited carrybacks and carryforwards of net operating losses, I.R.C. § 172(b)(1), but many taxpayers have losses far in excess of what they can utilize by these means. These restrictions would continue to apply even if there were no special limitations on capital losses. While these additional restrictions also make investing riskier, their existence is unrelated to the realization requirement.

19 I.R.C. § 1259(a), *added by* the Taxpayer Relief Act of 1997, Pub. L. No. 105-34, § 1001(a), 111 Stat. 768, 903–06.

lower rate when realized. At times when a capital-gains rate cut is anticipated, this kind of tax planning is literally front-page news.[20]

Most tax shelters over the years have been devices to defer tax, with conversion of ordinary income into capital gain often an important secondary goal. Real estate tax shelters achieve deferral primarily through the allowance of depreciation deductions that exceed actual economic depreciation in the value of the real estate; the excess is not reflected in income until a much later time, when the depreciation period is over or the property is sold. Straddle tax shelters achieve deferral through selective realization of losses. Many of the advantages of these tax shelters have been limited in recent years,[21] but the entire controversy surrounding tax shelters would never have arisen but for the realization requirement.

Planning opportunities remain. For example, owners of future interests in property can expect their interests to appreciate in value in a fairly predictable way as the date on which the intervening interest expires gets closer, but this appreciation is not taxed before the interest becomes possessory, or even at that time, because no realization event is considered to occur until the interest is disposed of.[22]

Sometimes deferral is allowed even after realization has occurred. The rules governing tax-free reorganizations,[23] like-kind exchanges,[24] involuntary conversions,[25] and installment sales[26] all per-

20 *See* Tom Herman, *Tax Report*, WALL ST. J., Sept. 6, 1995, at 1, col. 5; *see also* Allan Sloan, *Passing the Smell Test,* NEWSWEEK, Dec. 4, 1995, at 57 (discussing short sale used to the defer gain of the sale of Estee Lauder Inc. shares).

21 *See., e.g.,* I.R.C. § 469 (precluding losses from passive activities such as real estate investments from offsetting earned income and income from active businesses); § 1092 (restricting loss deductions from straddles until corresponding gains are realized).

22 *See* Noël B. Cunningham & Deborah H. Schenk, *Taxation Without Realization: A "Revolutionary" Approach to Ownership*, 47 TAX L. REV. 725, 759 (1992).

23 I.R.C. §§ 354–368.

24 I.R.C. § 1031.

25 I.R.C. § 1033.

26 I.R.C. §§ 453, 453A, 453B.

mit taxpayers to postpone the day of tax reckoning beyond the date of a realization event. These provisions are generally intended to mitigate the somewhat arbitrary consequences that a strict application of the realization requirement would entail.[27] Their effect, however, is to magnify the distortions of income measurement that the realization requirement creates.

f. Revenue Effects. The realization requirement costs the government money; the lost revenue has to come from someplace. To the extent that the revenue comes through higher income tax rates, the realization requirement intensifies the adverse effects that higher rates are thought to have: reduced incentives to work and invest, and a focus on tax factors rather than economic factors in business planning. The key to the tax reform of the 1980's was lowering tax rates while broadening the income tax base.[28] The realization requirement limits the potential reach of these efforts, because it constrains the income tax base.

g. Complexity. Several developments over the past two decades increased the importance of the realization requirement. First, a period of high inflation, followed by persisting high real interest rates, made the benefits of tax deferral more valuable.[29] This expanded the market for tax shelters, such as the straddle arrangements and real estate investments discussed above.[30] Second, governments allowed

27 For example, the reincorporation in Delaware of General Motors, formerly a New Jersey corporation, was technically a realization event, and in fact triggered a tax on the shareholders' gains even though they had in no substantive way altered their investment. *Compare* United States v. Phellis, 257 U.S. 156 (1921) and Marr v. United States, 268 U.S. 536 (1925) (reincorporation in a different state held to be a realization event) *with* Weiss v. Stearn, 265 U.S. 242 (1924) (reincorporation in the same state held not to be a realization event). The transactions in these cases predated the adoption of the reorganization rules. *See* I.R.C. § 368(a)(1)(F) (defining reorganization to include change in place of incorporation).

28 S. Rep. No. 99-313, at II-3 to II-8 (1986), *reprinted in* 1986-3 C.B. (vol. 3) 3–8; C. EUGENE STEUERLE, THE TAX DECADE 90–93 (1991).

29 Steuerle, *supra* note 28, at 34–37.

30 *See* text accompanying note 21.

interest and currency rates to fluctuate more freely in response to market conditions, increasing the demand for hedging arrangements such as interest rate and currency swaps, options, and futures contracts, and these financial instruments evolved rapidly to cover other types of risks.[31] These derivative instruments (so-called because the payoff is derived from the performance of something else) act as a universal solvent on the barriers that the tax law erects to classify income into various categories with differing tax treatment. Finally, the tax law struggled to deal with the new financial instruments in an intelligent way, and to curtail the proliferation of tax shelters.[32] While great progress was made towards these goals, a serious side effect was a level of complexity undreamt of twenty years ago.[33] Even back then, denouncing the complexity of the tax law was a campaign platitude.[34]

The entire corporate income tax can be seen to be a side effect of the realization requirement. If shareholders were taxed on the increase in the value of their shares, they would pay tax on corporate income, and there would be no need to tax the corporation. As long as corporate and individual tax rates are similar, the corporate tax on reinvested corporate profits captures the benefit to shareholders of deferring tax on those profits.[35] While in a rough way the corporate tax offsets some of the benefits of the realization requirement for corporate shareholders, it does so unevenly, and compounds the complexities of the realization requirement with all of the complexities that come from having a corporate tax itself.

31 *See* Warren, *supra* note 5, at 460.

32 *See id.* at 482–91.

33 Steuerle, *supra* note 28, at 155–60.

34 *See* JIMMY CARTER, WHY NOT THE BEST? 15 (1975).

35 ALI, Federal Income Tax Project, Subchapter C, Reporter's Study Draft 31–35 (1989).

2. *Why We Can't Live Without It*

Academics dream of an "accretion" tax, under which appreciation is taxed annually regardless of whether the asset is sold.[36] Doing away with the realization requirement is tempting. Yet, except in limited circumstances, an accretion tax is generally thought to be a pipe dream.[37] The two biggest obstacles are liquidity and valuation.

a. Liquidity. A law student in a Kurt Vonnegut novel is told that an alert lawyer would be "looking for situations where large amounts money were about to change hands."[38] This of course, is the "magic moment" in which a bit of this money can most easily pass into the hands of the lawyer. The realization requirement puts the Commissioner of Internal Revenue in the same position: she is there when the money changes hands. Under an accretion system, however, the tax is due each year on an asset's appreciation whether or not the asset is sold, and the tax must be paid in cash. There is no assurance that the taxpayer will have the money to pay the tax.

Perhaps if an accretion tax were the norm, people would plan their affairs to have the cash available when needed to pay taxes. Even under current law, taxable income can arise without a corresponding source of cash; indeed, most business taxpayers are required to report taxable income on the accrual basis, which deviates from the pattern of cash flows.[39] Paying taxes on income that has not yet been collected from customers is just another cost of doing business that must be financed, just like, say, inventory. Departures from the realization requirement exist for many types of pass-through entities; investors in

[36] See Shakow, *supra* note 7, at 1119; Theodore S. Sims, *Long-Term Debt, the Term Structure of Interest and the Case for Accrual Taxation*, 47 TAX L. REV. 313 (1992).

[37] *See, e.g.,* David Slawson, *Taxing as Ordinary Income the Appreciation of Publicly Held Stock*, 76 Yale L.J. 623 (1967).

[38] Kurt Vonnegut, Jr., GOD BLESS YOU, MR. ROSEWATER 9 (1965).

[39] I.R.C. § 448(a) (providing that the cash method is not generally available to a subchapter C corporation, a partnership with a subchapter C corporation as a partner, or a tax shelter).

these entities must pay taxes on their shares of the entity's income whether or not distributed.[40] These investors must make arrangements to receive cash distributions, or find other sources of cash to pay these taxes. Many financial instruments are marked to market or accounted for in ways that cause income to be taxed before the corresponding cash is received;[41] investors are on notice of these requirements before buying the instruments.

In theory at least, the taxpayers could also generate the cash by selling the assets with the gains that gave rise to the tax. Many types of assets are not easily sold, however, particularly interests in small, closely-held businesses.

Most taxpayers could adapt most of the time to the cash requirements of an accretion tax, but the exceptions would get the attention. The income tax is politically acceptable largely because it is based on ability to pay. This ability to pay depends on liquidity as well as net worth. A system that taxes increases in net worth without regard to liquidity will disrupt the affairs of taxpayers far more than a system that simply scoops up some of the cash when it appears at the time of sale.

b. Valuation. The bigger bugaboo is valuation. An accretion tax would require all assets to be valued at the end of each accounting period. The resulting administrative burden is thought to be prohibitive.[42]

Valuation is fundamental to some non-income taxes. Estate taxes are imposed on the fair market value of estates,[43] and property taxes are assessed annually based on fair market value.[44] These taxes, however, have features that make the administrative burden of valua-

40 *See, e.g.,* I.R.C. § 702 (providing flow-through treatment for partnerships).

41 *See, e.g.,* I.R.C. § 1272 (providing for taxation of accrued original issue discount).

42 Simons, s*upra* note 6, at 56, 82–89, 100, 103–04; Andrews, Cash Flow Tax, *supra* note 6, at 1141–42;.

43 Treas. Reg. § 20.2031-1(b).

44 *See, e.g.,* N.Y. REAL PROP. TAX LAW §§ 300–302 (McKinney 1984 & Supp. 1997).

tion more tolerable. Estate taxes are imposed on a limited class of wealthy taxpayers, and on each of these only once. Moreover, the tax is imposed on a collection of assets that is typically a windfall of sorts to the beneficiaries. While the beneficiaries themselves may care deeply about how the estate is valued, the rest of us can be content with rough justice.

Property taxes typically apply only to real estate within a given community. While assessments are made annually, they do not need to track absolute values accurately, provided that they track relative property values within the community. These relative values are unlikely to change dramatically from year to year. Every so often, the assessments need to be reviewed, and opportunities for challenge provided. But the limited geographical scope of the property sites makes it easier to arrive at valuations that are fair relative to each other.

When valuation issues arise under the income tax, disputes between the taxpayer and the Internal Revenue Service degenerate into a battle of the experts.[45] The importance of appraisers is so great that the value of an asset is seen as whatever an appraiser can defensibly say it is.

Valuation is generally viewed as a practical problem. Under this view, each asset has its worth; it is just sometimes hard to know what it is. With enough information, and sufficiently sophisticated analysis, one could zero in on the "true" value of any asset. Yet there is a deeper problem with valuation. The very idea assumes that for each asset there is some objective number that represents its value. It may be hard to find out; it might even be unknowable; but it is there.

This objective concept of value is hard to sustain in light of how value is conventionally defined: the amount that a willing buyer would pay a willing seller, each being under no compulsion to buy or

[45] E.g., Fed-Mart Corp. v. United States, 75-2 USTC ¶ 9531 (S.D. Cal. 1975).

sell and each having knowledge of all relevant facts.[46] This definition is a counterfactual: it is based on what hypothetical people *would* do, not on what actual people are actually doing.[47] Except for assets that are publicly traded or can be readily converted to cash (which are the only easy cases), valuation is an exercise in make-believe. Valuation is determining what people would pay for an asset if there were actually someone out there to buy it and someone else willing to sell it.

To describe an object in terms of what will happen in hypothetical circumstances is to assign a *disposition* to that object.[48] In the classic example, to say that sugar is soluble in water is to say of any lump of sugar that it *would* dissolve if it *were* placed in water. Similarly, to say that an asset is worth $100 is to say that it *would* fetch a price of $100 if it *were* put up for sale. What if a particular lump of sugar is never placed in water, or the asset is never sold? In the case of the sugar, we can call it soluble with some confidence based on our observation of similar lumps of sugar in similar circumstances. Ultimately, however, the real explanation comes not in terms of some mysterious "disposition" to dissolve, but rather in terms of a scientific theory having to do with polar bonding in water and sugar molecules.[49]

Valuing an asset can be done with some confidence if similar assets are actually being sold, although no two assets or circumstances of sale are ever quite identical.[50] In the absence of a comparable sale, what is needed is some means of analyzing the asset in order to de-

46 Treas. Reg. § 20.2031-1(b) sets forth this general rule for valuing property for estate tax purposes. This same rule is used in the income tax regulations. *See, e.g.,* Treas. Reg. 1.148-5(d)(6)(i).

47 For a discussion of the philosophically suspect status of such counterfactual conditionals, *see* NELSON GOODMAN, FACT, FICTION AND FORECAST 3–27 (4th ed. 1983).

48 *See generally* ELIZABETH W. PRIOR, DISPOSITIONS (1985) (discussing problems that arise in making a dispositional statement about an object).

49 *See* WILLARD V.O. QUINE, THE ROOTS OF REFERENCE 8–12 (1973).

50 The author's home was valued some time ago by two appraisers in connection with applications for a home equity loan. Each appraiser used a comparable sales approach, but one appraisal was nearly 50% higher than the other.

duce its value, much as chemical theory can be used to deduce the solubility of sugar in water. Yet one cannot take an asset apart and look at its "value." Common appraisal methodologies, such as income forecast and replacement cost, represent attempts find comparable sales either of the goods or services that the asset produces, or of its component parts. Yet the income forecast is speculative, because it seeks to determine what an asset will earn in the future, a time when no comparable sales data are available. The replacement cost approach is limited, because assumes that the asset is no more than a collection of fungible parts each of which itself has a readily ascertainable market value. In the end, no such analytical approach can assign a "true" value to an asset, because value is not an intrinsic property of an asset, but rather represents the price that people will pay for it. This price depends as much on the external demand for the benefits that the asset provides, as well as on the availability of substitutes, as it does on the particular attributes of the asset itself.

Rather than say that an asset as a particular value, it would be better to say that it has a spectrum of values, representing the possible range of outcomes if it were put up for sale. In some cases, such as publicly traded stock, the range might be quite small, although the valuation of large blocks of such stock raises difficult questions of blockage and control.[51] In most cases, however, the range is quite significant, and some appraisers recognize this point by specifying

[51] Blockage refers to the difficulty of marketing a large block of stock, because dumping such a quantity of stock on the market at one time can significantly depress the price. Indeed, a principal function of stock underwriters is to provide assurance that such a block of stock can be marketed. Control issues can affect valuation because a block of stock that carries with it the practical ability to control a corporation is generally thought to be worth more for that reason. *See* Estate of Salisbury v. Comm'r, 34 T.C. Memo (CCH) 1441, 1451 (1976). Similarly, minority blocks of stock that lack such control may be subject to a discount for that reason, although the analysis gets quite tricky when the possibility of coalitions and "swing" blocks of stock is introduced. *See* Estate of Winkler v. Comm'r, 57 T.C. Memo (CCH) 373 (1989); I.R.S. Tech. Adv. Mem. 94-36-005 (May 26, 1994).

their conclusions in the form of a range. There is an indeterminacy about value that is analogous to the indeterminate position in space of a subatomic particle under quantum theory. Prior to observation, its position is represented by a wave function that is related to the probability of the particle's being at each point in space.[52] Upon detection, the wave function "collapses," and the particle is assigned to the location where it is detected with 100% probability.[53] Similarly, the range of potential value of an asset collapses to a single number when the asset is sold. Until such a sale, however, the value is indeterminate. Indeed, the indeterminacy is even worse than in quantum theory, where the probability distribution can be specified with precision.[54] In an appraisal, even the range of outcomes, and their respective probabilities, is a matter of speculation.

A valuation process, perhaps conducted under canons of valuation developed by the appraisal community, can serve a useful function to the legal system by coming up with numbers that can be used to fix tax liabilities and thereby resolve disputes. We should not, however, expect more of this process. It is not merely that the process may produce inaccurate results in particular cases; it is that there is no standard against which the process can be said to be accurate or not.

These considerations weaken one's faith in the Haig-Simons definition of income. Defining income by reference to changes in value,[55] when the values are not well defined *even in theory*, yields a definition of income that is not well defined *even in theory*. Yet the very purpose of the Haig-Simons definition is to serve as a theoretical benchmark, against which actual schemes of computing income can be assessed.

52 Richard P. Feynman, Robert B. Leighton & Matthew Sands, THE FEYNMAN LECTURES ON PHYSICS § 37-4 (Commemorative issue 1989).

53 *Id.*; R.I.G. Hughes, THE STRUCTURE AND INTERPRETATION OF QUANTUM MECHANICS 226–28 (1989).

54 Feynman et al., *supra* note 52, at § 37-4; Hughes, *supra* note 53, at 226–28.

55 *See supra* text accompanying note 6.

Thus, the problem with accretion taxation is not simply that it is hard to figure out the proper amount of taxable income. There simply is, at bottom, no proper amount of taxable income to figure out.

B. *What We Can Do About It*

The tax law uses three methods in particular contexts to cut back the scope of the realization requirement. The first method is to require unrealized gains and losses to be reported annually. While this method is the most straightforward, it is currently used only in contexts where the liquidity and valuation problems are thought to be manageable. The second method is to use concepts of accrual accounting to redefine when realization takes place. This method avoids valuation problems because income is determined under these accounting concepts rather than by asset valuation. The third method is to charge interest on the tax liability that is deferred by reason of the realization requirement. This last method avoids liquidity problems because the tax is not due until a realization event occurs.

1. *Mark to Market*

Under a mark to market system, gains and losses are taken into account each year whether realized or not. On the last day of the year, each asset that has not been sold is "marked to market" by treating it as if it had been sold on that day at a price equal to its fair market value, and then reacquired at the same price.[56] The tax law uses mark to market sparingly, because of the liquidity and valuation problems that such a wholesale abandonment of the realization requirement entails.

The first such use of mark to market appeared in the Economic Recovery Tax Act of 1981, which attacked the use of straddles to

[56] *E.g.,* I.R.C. § 1256(a)(1).

defer tax liability.[57] A "straddle" is a combination of short and long positions in the same commodity or financial asset in which a loss on position is expected to be offset by gain on the other.[58] These positions are often acquired by means of forward contracts, which are contracts to buy (in the case of a long position) or sell (in the case of a short position) a fixed amount of the underlying commodity or financial asset at a given date in the future. The holder of a straddle is not exposed to fluctuations in the price of the underlying asset, because any increase in the value of the long position will be offset by a decrease in the value of the short position, and vice versa. By closing (and re-establishing) a loss position while keeping the offsetting gain position open, the holder can generate a tax loss that does not correspond to any economic loss. While a corresponding taxable gain would be reported in the subsequent year in which the gain position is closed, this gain can be sheltered in that year by triggering a loss on yet another straddle.

These straddle techniques are now foreclosed by rules that require a straddle loss to be deferred until gain on the offsetting position is recognized.[59] In the case of futures contracts, however, the tax law addresses straddles by means of a mark to market system rather than loss deferral.[60] Futures contracts are forward contracts that are established through a commodities exchange and regulated by the Commodity Futures Trading Commission. These contracts are subject to public trading, and each day's gain or loss can be readily determined by reference to trading prices. The holder of a futures contract is required to deposit with a broker as margin an amount equal to that day's loss on a futures contract, and is entitled to recover from the broker an amount equal to that day's gain. This system is itself known

57 Pub. L. 97-31, § 503(a), 95 Stat. 172, 327–30 (1981).

58 *See* I.R.C. § 1092(c).

59 I.R.C. § 1092(a).

60 I.R.C. § 1256.

as "marking to market," and causes holders of these contracts to realize cash inflows and outflows even while the position remains open.

The realization requirement is meaningless for futures contracts that are marked to market, because the exchange rules impose a sort of realization event on a daily basis. It was therefore a short step for the tax law to impose a mark to market regime for reporting gains and losses on these contracts. This approach was subsequently extended to options on futures contracts, which are subject to similar exchange rules. [61]

For these types of contracts, doing away with the realization requirement poses no dilemma. No liquidity problem exists because margin receipts can free up the funds with which to pay the tax. No valuation problem exists because public trading prices are used to determine these margin payments. Restricted to this context, however, the mark to market rule makes little headway against long-term deferral, because these options and futures tend to be of short duration, typically less than one year.

The mark to market regime was further extended in 1993 to the inventory of securities dealers.[62] Here again, liquidity and valuation problems are minimal. The dealers make a market in these securities; this activity generates ascertainable market values and also provides a source of liquidity.

2. *Accrual of Income*

In a sense, the entire accrual method of accounting is an affront to the realization requirement. Semantically, the issue is suppressed by redefining the realization event to be something other than the receipt of cash or other property. An accrual basis seller recognizes income at

61 I.R.C. § 1256(b), *as amended by* the Deficit Reduction Act of 1984, Pub. L. 98-369, § 102(a)(2), 98 Stat. 620–22.

62 I.R.C. § 475, *added by* the Omnibus Budget Reconciliation Act of 1993, Pub. L. 103-66, § 13223(a), 107 Stat. 312, 481–84.

the time of sale, rather than the time of payment.[63] Periodic items such as rent and interest are reported as earned, rather than when paid.

Liquidity problems arising from the erosion (or redefinition) of the realization requirement are accepted without complaint. In most cases, the accrual of income and the related cash receipt are much less than a year apart. The accrual of deductions for expenses in advance of payment[64] reduces the net burden and at times can create a net benefit. Most businesses treat tax liability on accrued income as simply another component of working capital that needs to be financed. The burden is real, however, and businesses, such as law firms, that are entitled to use the cash method of accounting have significantly lower financing needs as a result.

Valuation problems under accrual accounting are dealt with by a set of formal rules that substitute for actual valuations. These rules are not intended to be an accurate substitute for valuation; they merely provide a means of reporting income in a consistent and objective manner. In most cases, the rules are straightforward: short-term receivables and payables arising from accrued items of income and expenses are taken into account at face value. Long-term receivables for the sale or use of property are taken into account on a present value basis using a discounting formula.[65] While in principle a similar approach could be applied generally to expenses that accrue long be-

63 *See* Treas. Reg. § 1.451-1(a) ("Under an accrual method of accounting, income is includible in gross income when all the events have occurred which fix the right to receive such income and the amount thereof can be determined with reasonable accuracy.").

64 A deduction accrues when all events have occurred that establish the fact of liability, the amount of liability can be determined with reasonable accuracy, and economic performance has occurred. Treas. Reg. § 1.461-1(a)(2). Economic performance can occur in advance of payment. I.R.C. § 461(h)(2).

65 Treas. Reg. § 1.1001-1(g)(1); Prop. Treas. Reg. § 1.467-2, 61 Fed. Reg. 27,834, 27,842–44 (June 3, 1996).

fore payment,[66] the tax law instead allows a deduction for the full amount but defers the deduction until the year when "economic performance" occurs, which is typically the time of payment, or not long before.[67]

a. Accrual on Debt Instruments. The scope of accrual has been extended to prevent cash-basis taxpayers from deferring tax on income from long-term debt instruments issued at a discount from their face value. The holder of such an instrument can realize gain equal to the amount of this income, irrespective of market fluctuations, simply by holding the instrument until maturity.[68] This gain is equivalent to interest income, and is required to be taken into account over the term of the obligation on a basis that reflects the accrual of this income at a constant interest rate.[69]

Accrual of discount on debt instruments differs from the mark to market method because there is no assurance that the purchase price plus accrued discount will be equal to the fair market value of the instrument at any point in time before maturity. Debt instruments can fluctuate in value with changes in the perceived creditworthiness of the issuer, or with changes in prevailing interest rates.[70] While the accrual of discount fails to reflect accurately unrealized gains and

66 S. Prt. No. 169, Vol. I, 98th Cong., 2d Sess. 266–67 (1984); H.R. Rep. No. 98-432, Pt. 2, at 1254–55 (1984).

67 I.R.C. § 461(h).

68 Gain is equally assured when an instrument issued at par is acquired at a discount on the secondary market, but accrual of this market discount is elective. I.R.C. § 1278(b).

69 I.R.C. § 1272(a).

70 Even if the creditworthiness and interest rates remain perfectly static, accrual of discount will not properly reflect growth in value. If, as is usually the case, long-term interest rates are higher than short-term rates, a long-term instrument will increase in value more than the amount reflected by the accrual of discount as the maturity date grows closer, because over time it becomes a short-term instrument but its yield to maturity reflects a long-term rate. After a point, the valvalue will begin to grow less rapidly, because the advantage of an above-market rate declines as the maturity date grows even closer. *See* Sims, *supra* note 36, at 315–17.

losses, it does a far better job than would a strict application of the realization requirement, which would defer taxation of discount until maturity.

Accrual of discount on fixed debt obligations has appeal because the amount of income that accrues over time can be determined with certainty. The application of accrual accounting is far more problematic when the amount due on a debt obligation is contingent. The Treasury Department's first attempt to write regulations governing contingent debt obligations provided for deferral of the holder's income (and the issuer's deductions) until the contingencies were resolved.[71] This approach had most of the virtues and faults of the realization requirement itself. Its principal virtue was its objectivity: there was no need to speculate about how a contingency might be resolved, because there were no tax consequences while the contingency was open. This treatment of contingent items, however, created unintended opportunities for holders of contingent debt instruments, and unintended hardships for issuers. For example, issuers of debt with fixed interest but contingent principal could not deduct any interest payments until it became clear that total payments on the instrument would exceed its issue price.[72] Similarly, holders of such instruments could exclude these interest payments from income, effectively treating them as a return of principal.

Responding to strong criticism of the initial contingent debt regulations,[73] the Treasury developed new rules for accruing interest on contingent debt instruments based on estimates of how the contingency will be resolved. These estimates are based either on market information regarding the contingency, or if this information is not available, on the assumption that such an obligation will accrue interest at a rate at least equal to the rate on the issuer's non-contingent

71 Prop. Treas. Reg. § 1.1275-4(c)(3)(i), 51 Fed. Reg. 12,022, 12,087 (Apr. 8, 1986).

72 Prop. Treas. Reg. § 1.1275-4(f), 51 Fed. Reg. 12,022, 12,092 (Apr. 8, 1986).

73 *See* DAVID C. GARLOCK, FEDERAL INCOME TAX OF DEBT INSTRUMENTS 6-54 to 6-57 (3d ed. Supp. 1997) (discussing the 1986 and 1993 proposed regulations).

debt.[74] These rules go far beyond spreading a known quantity of income over the holding period to which it relates. They *assume* that a given amount of income has been earned in each period, even though this income may never be realized if the resolution of the contingencies is more adverse than estimated. In the end, the system is self-correcting: any deviation between estimates and actual events is ultimately corrected by an appropriate amount of gain or loss that is reported when the contingency is resolved.[75] As always with the realization requirement, timing is everything.

Taxing holders of contingent debt instruments on assumed income that may never even be reflected in market value, let alone be realized, sounds harsh, and it is harsh. The promise of an offsetting loss deduction at some later date is cold comfort. Yet even taxpayers with actual, realized gains, must pay tax on those gains even though the gains may be wiped away by subsequent losses.[76] Some ancient Greeks refused to call a man happy before his death, for who knew what misfortunes lay ahead?[77] This view has found its way into tax scholarship in the form of Vickrey's proposal for lifetime income averaging.[78] As long as income is determined, and taxes paid, on a periodic basis, taxpayers will be taxed on accessions to wealth that, with hindsight, prove to have been short-lived. At least the accession to wealth, however, temporary, reflects the taxpayer's ability to pay *at that time*. By contrast, the recognition of income by the holder of a

74 Treas. Reg. § 1.1275-4(b)(4).

75 Treas. Reg. § 1.1275-4(b)(6)(i).

76 "[T]hat gains may be succeeded by losses can hardly be taken to show that the taxpayer has not been enriched when the value of his property appreciates." MARVIN CHIRELSTEIN, FEDERAL INCOME TAXATION 72 (6th ed. 1991) (emphasis deleted).

77 Plutarch, *Solon*, in PLUTARCH'S LIVES 114 (John Dryden, trans., Modern Library ed. 1932).

78 William Vickrey, *Tax Simplification Through Cumulative Averaging,* 34 LAW & CONTEMP. PROBS. 736 (1969); William Vickrey, *Averaging of Income for Income-Tax Purposes,* 47 J. POL. ECON. 379 (1939).

contingent debt instrument during a period when the holder has in fact suffered a market value loss is a serious burden compared with the allowance of a loss deduction under the Haig-Simons ideal.

With these contingent debt regulations, the triumph of accrual accounting of debt instruments is nearly complete.[79] By contrast, stock investments are generally accounted for on a cash basis. It is no accident that the classic statement of the realization requirement dates back to *Eisner v. Macomber,*[80] a case involving stock dividends. Dividends are taxed when declared and paid, even if the stock is preferred stock with a fixed dividend rate and the holder is an accrual basis taxpayer.[81] The holder of preferred stock issued at a discount below its par value, however, must accrue the discount as a dividend over the term of the preferred stock.[82]

b. Pass-Through Entities. Accrual concepts override the realization requirement for stock holdings in corporations that are treated as pass-through entities for Federal income tax purposes. For example, the holder of stock in an S corporation must report annually the holder's share of the corporation's income and loss.[83] This share of income may have no particular relation to the value of the holder's stock investment, and, so far as the holder is concerned, no realization event has occurred. In effect, the shareholder has consented to waive the benefits of the realization requirement in order to avoid corporate-level taxation. Such an express election on the part of each

79 The principal exception is that cash basis taxpayers can report stated interest when paid, provided that the interest is payable regularly at intervals of one year or less. Treas. Reg. §§ 1.61-7(a), 1.446-2(a)(1), 1.451-1(a), 1.1272-1(a)(1), 1.1273-1(c). A few minor exceptions also exist in § 1272(a)(2).

80 252 U.S. 109 (1920).

81 *See* Treas. Reg. § 1.61-9(c).

82 I.R.C. § 305(c)(3) (applying principles of § 1272(a)).

83 I.R.C. § 1366(a). Similar requirements exist for capital gains that a regulated investment company elects to treat as having been distributed. I.R.C. § 852(b).

shareholder is a condition to treatment of the corporation as an S corporation.[84]

Income of a foreign subsidiary of a United States parent is generally not subject to United States tax until dividends are paid to the parent. To prevent unwarranted deferral of tax, some categories of income of these subsidiaries must be reported by the parent when earned by the subsidiary.[85] This form of pass-through taxation is mandatory, not consensual; the subsidiary's realization events serve as a proxy for the parent's.

3. *Interest on Deferred Tax*

Mark to market is confined to limited spheres in which liquidity and valuation concerns are absent. Accrual accounting works well for predictable types of income, but is much more problematic when applied to contingent returns. A third approach respects the realization requirement but seeks to compensate for the effects of deferral. Under this approach, the income ultimately realized is allocated over the years in the holding period according to a formula. The additional tax that would have been due in each such year on that year's income is determined, and is payable in the year of realization together with interest over the interval between the year to which the income was allocated and the year in which the tax is paid. The interest charge approach meshes perfectly with the idea of tax deferral as an interest-free loan from the government. If the problem is the interest-free nature of the loan, the solution is to charge interest.[86]

84 I.R.C. § 1362(a)(2).

85 I.R.C. § 951(a).

86 Many have advocated this approach. *See, e.g.,* Blum, *supra* note 3, at 7–12; Roger Brinner, *Inflation, Deferral and the Neutral Taxation of Capital Gains,* 26 NAT'L TAX J. 565, 570–71 (1973); Roger Brinner & Alicia Munnell, *Taxation of Capital Gains: Inflation and Other Problems,* NEW ENG. ECON. REV. Sept.-Oct. 1974, at 3, 12–21; CONG. BUDGET OFFICE, REVISING THE INDIVIDUAL INCOME TAX 78–81 (1983); INSTITUTE FOR FISCAL STUDIES, THE STRUCTURE AND REFORM OF DIRECT TAXATION 132–35, 148–49; Fellows, *supra* note 3, at 737 *et seq.*; John

The tax law has only occasionally used this interest charge approach. The most noteworthy example is the treatment of private foreign investment companies (PFICs). A PFIC is a foreign corporation with a significant portion of its assets being of a type that generate passive income such as dividends, interest, and royalties.[87] PFICs are generally permitted to accumulate income tax-free, but upon a realization event (such as a sale of stock or an extraordinary dividend) the realized income is allocated over the shareholder's holding period on a ratable basis.[88] The amount of income allocated to each prior year is subject to tax at the highest marginal rate then in effect for that year, and an interest charge is computed from the prior year to the year the income is realized.[89] The interest charge is based on a rate equal to three percentage points over the average market yield on United States Treasury securities for each calendar quarter, and is compounded daily.[90]

The interest charge approach is also used for the taxation of "accumulation distributions" from foreign trusts.[91] An accumulation distribution is a distribution of a prior year's trust income. Like PFICs, foreign trusts can be used to accumulate income offshore on a tax-free basis. The tax on accumulation distributions seeks to recover this tax when the income is finally distributed to the beneficiary, together with interest from the years in which the income was earned. The tax is determined at a rate intended to approximate the beneficiary's aver-

Helliwell, *The Taxation of Capital Gains,* 2 CAN. J. OF ECON. 314 (1969); INSTITUTE FOR FISC. STUD., THE STRUCTURE AND REFORM OF DIRECT TAXATION 132–35, 18–49 (1978) (Meade Report); James W. Wetzler, *Capital Gains and Losses,* in COMPREHENSIVE INCOME TAXATION 115, 152–53 (Joseph A. Pechman ed. 1977).

87 I.R.C. § 1297(a).

88 I.R.C. § 1291(a).

89 I.R.C. § 1291(c).

90 I.R.C. § 6621.

91 See I.R.C. § 668(a).

age marginal tax rate over the deferral period,[92] and interest is charged at the tax underpayment rate over the deferral period.[93]

The interest charge approach has been touted as a no-fuss solution to the dilemma posed by the realization requirement.[94] It does solve the liquidity problem, because no tax is due until the realization event occurs. The valuation problem remains, however, and the interest charge approach can significantly over- or under-correct for the benefits of tax deferral in particular cases.[95]

The problems with the interest charge approach flow from its view of tax deferral as an interest-free loan from the government. Under this view, the tax that would have been paid under an accrual system, but which was not paid because of the realization requirement, is considered to have been loaned by the government to the taxpayer, and repaid at the time of the realization event. Current law, of course, does not charge interest on this deemed loan; the taxpayer gets the use of the government's money for free. Any proposal to charge interest on the deferred tax must address two questions. First, what is the principal amount of the deemed loan? Second, what should be the interest rate?

a. Principal on the Deemed Loan. The principal amount of the deemed loan is the tax that should have been paid but was not. The tax that should have been paid is, ideally, the tax that would have been imposed under a system of accretion taxation. In each year, the deemed loan goes up (or down) by the tax rate multiplied by that year's unrealized increase (or decrease) in value. Computing the deemed loan in this fashion, however, resurrects the valuation problem, because the unrealized increase or decrease in value can be determined only by valuing the asset at the end of each year.

92 I.R.C. § 667(b).

93 I.R.C. § 668(a).

94 *See* authorities cited in note 86 *supra.*

95 *See* Cunningham and Schenk, *supra* note 22, at 745.

To sidestep the valuation problem, interest charge proposals typically rely on an accounting formula to determine each year's accrued income. In effect, the asset is deemed to have the value at the end of year that is determined by the formula. For example, the PFIC rules assume that income realized upon a sale or extraordinary distribution was earned ratably on a straight-line basis over the holding period.[96] This assumption is simplistic and arbitrary, but it could be defended as better than current law, which assumes that no income arises until the last moment.

The PFIC rules could be refined. Straight-line accrual is more rapid than accrual at a constant rate, because of the effects of compounding. Simply stated, over the term of the investment any undistributed earnings generate income after they are reinvested, so one would expect, if the rate of return were constant, that larger amounts of income would be earned towards the end of the holding period than towards the beginning. The straight-line approach is biased in that it assumes more tax deferral than would occur if income had been earned at a constant rate. Investors recognize this bias, and the PFIC rules are widely believed to over-compensate for the benefits of deferral.[97]

Another way to estimate the PFIC shareholder's income for each year is by reference to the shareholder's share of the PFIC's income in that year. The practical difficulty with this approach is that the PFIC is normally outside United States jurisdiction, and therefore is not required to report its income to the IRS. The shareholders themselves may have no way of knowing what the annual income is,

[96] I.R.C. § 1291(a). This straight line allocation is favored by Shakow, *supra* note 7, at 1122, but he ultimately rejects the interest charge approach in favor of a pure accretion system, mainly because of concerns about the appropriate tax rate to impose in a case where the rate has varied over the holding period. This tax rate problem is discussed *infra* at Part V.A.4 (p. 362).

[97] *See* JOSEPH ISENBURGH, INTERNATIONAL TAXATION ¶ 44.16 (2d ed. 1996); Philip T. Kaplan, *Using PFIC's Offensively, in* ESSAYS ON INTERNATIONAL TAXATION 213, 215 (Herbert H. Alpert & Kees van Raad, eds. 1993).

or the reporting to shareholders may be done in a way that is inconsistent with United States tax accounting principles. The tax law allows a PFIC to elect to report its income, and in such a case the shareholder can elect to have the interest charge computed in just this way.[98] Alternatively, the shareholder can elect to avoid the interest charge altogether by paying tax each year on his or her share of the PFIC's income.[99]

A refined formula for determining each year's deferred tax would use statistical techniques to compute a weighted average of all possible paths of values that the asset might have had over the holding period.[100] When only the initial purchase price and final selling price are known, the intermediate values along the way cannot be known with certainty, but some values are more likely than others. This refinement, however, requires some statistical information on the volatility of the asset's market value.[101] This information is least likely to be available in precisely those cases where a mark-to-market approach is least likely to work: when the asset is unique or is otherwise not publicly traded.

b. Interest on the Deemed Loan. The second question that needs to be addressed by an interest charge approach is the appropri-

98 I.R.C. § 1294.

99 I.R.C. §§ 1291(d), 1293(a). The 1997 Act added a provision that permits shareholders of publicly traded PFICs to elect a mark to market approach in lieu of the interest charge or a pass-through of the PFIC's income. I.R.C. § 1296(a), *added by* the Taxpayer Relief Act of 1997, Pub. L. No. 105-34, § 1122(a), 111 Stat. 88, 972. As a result, some PFIC shareholders will have a choice of all three methods of reducing the deferral benefits of the realization requirement: mark to market, income accrual, and the interest charge.

100 *See* Strnad, Periodicity, *supra* note 15, at 1868–79, 1893–99.

101 This approach is analogous to the manner in which reflected light, under quantum electrodynamics, gets from point A to point B: not in a straight line, as supposed in classical physics, but through a weighted average of all possible paths. *See* Richard B. Feynman, QED: THE STRANGE THEORY OF LIGHT AND MATTER 38–49 (1985). An important difference is that in valuation estimates the probability distribution is a substitute for reality; but in quantum mechanics, the probability distribution *is* the reality.

ate interest rate. The choice of rate depends on whether one's focus is on keeping the government whole or on taking away the taxpayer's windfall. The cost to the government of tax deferral is essentially its borrowing cost. This cost is somewhat overstated, however, because taxpayers who paid tax on unrealized gains would have less to invest (or greater borrowing costs), which would reduce their tax liabilities and therefore reduce the revenues to the government. Deferral thus actually produces a revenue gain to the government, although not in amounts sufficient to compensate it fully for the increased borrowing costs. Quantifying the net cost is no simple matter, involving as it does assumptions or information regarding the after-tax rates of return, or avoided borrowing costs, of taxpayers who have more funds to invest precisely because of deferral. Another way to look at the net cost is to assume that the purchasers of the additional Treasury securities that need to be issued because of deferral are themselves subject to Federal income tax. Based on this assumption, the net cost to the government is its after-tax borrowing rate: if the stated rate is 10% and the tax rate is 35%, then the after-tax rate is 6.5%. If the interest rate were to be based on keeping the government whole, no doubt such an assumed rate would be used, if only to avoid the hopelessly difficult empirical challenge of implementing anything more precise.

Even if the government's net cost of deferral could be more precisely determined, one has to wonder whether the exercise would be worth the trouble. Setting the interest rate to keep the government whole takes the focus away from many of the drawbacks that make the realization requirement itself such a mixed blessing: horizontal inequity, vertical inequity, and distorted economic behavior.[102] To make the tax system more neutral and just from a taxpayer perspective, attention needs to be given to the benefits of deferral from the taxpayer's point of view.

[102] *See supra* Part II.A.1 (p. 281).

A taxpayer who defers tax can been seen as having either more funds to invest or a smaller need to borrow. If one assumes that tax deferral enables the taxpayer to avoid borrowing, the appropriate interest charge would be at the taxpayer's borrowing rate. This rate will, of course, vary from taxpayer to taxpayer, because more creditworthy taxpayers can borrow at a lower rate. If the interest charge approach respects these differences, then the interest charge will be greater for those whose credit is weak. While this makes sense from a lender's point of view, it has a strongly regressive air about it: the rich pay less than the poor.

There is also the administrative burden of figuring out each taxpayer's avoided borrowing cost. For a corporate taxpayer, the picture is further complicated by the cost of equity capital; few corporations can satisfy all of their financing needs with debt, because too much debt creates an unacceptable risk of bankruptcy. Equity capital has a far greater after-tax carrying cost than debt, because equity investments generally yield a higher rate of return (in order to compensate for the higher degree of risk), and the issuer cannot deduct the payment of this return. For example, it would not be unusual for a corporation to issue debt with a 6% return and equity with a 16% return (including retained earnings as well as dividends). The corporation can deduct interest on the debt, so the after-tax cost of 6% debt would be about 4%. But it cannot deduct dividends on the equity or retained earnings, so the after tax cost of the equity is also 16%. A 4-to-1 difference in the marginal costs of debt and equity capital is too great to be ignored, but what blend of debt and equity a taxpayer might have used to finance its tax liability on unrealized gains is a subtle question with no objective answer. Because of these difficulties of measurement, any interest charge approach based on avoided costs of capital is likely to apply a uniform rate to all taxpayers. For example, the interest charge under the PFIC rules is set at three points above the Treasury's borrowing rate for the same peri-

od;[103] the three-point differential presumably reflects the difference in borrowing rates for the Treasury and a typical taxpayer.[104]

Instead of reducing borrowing, a taxpayer who defers tax might choose to reinvest the funds not used to pay tax. With this focus, the appropriate charge for tax deferral could be based on the earnings of these reinvested funds. There is an obvious tracing problem here: which investments were bought with the funds that otherwise would have been used to pay tax? One could take a global approach, and assume that a pro rata portion of all of the taxpayer's investments can be attributed to deferred taxes, and charge interest on the deferred tax based on the overall return on the taxpayer's investments. Such a determination of overall return would require periodic valuations. These valuations would be necessary anyway if the *amount* of deferred tax were determined under a pure accretion system. If, however, the implementation used, like the PFIC rules, a formula to allocate the return among years in the holding period,[105] these valuations would not be needed for that purpose, and requiring these valuations in order to determine the appropriate interest charge would be an additional administrative burden.[106]

The taxpayer's investment return on the deferred tax might be determined by reference to the specific investment that generated the deferred tax. This approach has been thought to present the same valuation problems that would be involved in measuring the overall return,[107] although to determine the tax liability for a particular asset, only periodic valuations of that asset would be necessary. As shown in the next Part, this concern is misplaced: the appropriate adjustment for deferral can be done without periodic valuations. A more funda-

103 I.R.C. §§ 1291(c)(3), 6621(a)(2).

104 S. Rep. No. 99-313, at 184 (1986); H.R. Rep. No. 99-426, at 849 (1985).

105 I.R.C. § 1291(a).

106 This burden, however, is more manageable than might at first appear. *See infra* Part IV.C.2 (p. 348).

107 Blum, *supra* note 5, at 14; Warren, *supra* note 5, at 478.

mental objection to this approach is that it is inconsistent with the idea of tax deferral as an interest-free loan from the government. After all, it is a "loan" the repayment of which is dependent on subsequent profits. If the asset subsequently goes down in value, consistency would require the "loan" to bear a negative interest rate. If a commercial investment had these attributes, it would almost certainly be characterized as equity for tax purposes. This is not necessarily a real objection; after all, perhaps the concept of tax deferral as an interest-free *loan* should be revisited, and indeed will be revisited in the next Part. In the meantime, an approach that purports to charge "interest" based on the taxpayer's own investment results is best regarded as not an interest-charge approach at all.

It is likely that any interest-charge approach would use a uniform rate for all taxpayers in any particular year, based on the government's borrowing cost or some premium above that cost to reflect average private borrowing rates. Such an approach, when combined with a formula allocation of the finally realized gain over the years in the holding period, produces a means of calculating the tax without any need for periodic valuations. Since the tax is imposed only upon a disposition of the investment, there is no liquidity problem, either. Has the realization dilemma been solved?

c. Problems with the Interest Charge Approach. An interest-charge approach would be a vast improvement over the present system. It would offset, in a crude way, the benefits of deferral, without the liquidity and valuation problems that an accretion tax would create. Yet the approach is not without its own significant problems, which can be illustrated by the use of this approach in the PFIC context. As noted above, PFIC investors regard this method as somewhat punitive in its effects.[108] To some degree, *any* approach that offsets the benefits of tax deferral will be seen as punitive, if only because investors are used to deferral, and have come to see it as a birthright. Yet

[108] *See supra* note 97 and accompanying text.

these investors have a point. If, as is the case with the PFIC rules, the formula for allocating income among the years of the holding period is front-loaded compared with accrual at a constant rate, then the interest charge will tend to be overstated. Even if the formula were adjusted to reflect accrual of income at a constant rate, the formula will under-correct for the benefits of deferral income in cases where most of the gains occur early in the holding period, and will over-correct for these benefits where the gains occur late.

These under- and over-corrections will prompt tax-motivated behavior. Suppose, for example, an investor realizes above-normal returns shortly after investment. The investor will have an incentive to hang on to that investment in order to "average down" the overall return by the time the investment is disposed of. The PFIC formula will have the effect of smoothing out the high initial return over the balance of the holding period, so the correction for tax deferral will be smaller because the assumed length of deferral is shorter than was actually the case. The opposite tendency will occur when an investment realizes below-normal returns early in the holding period. Here, the investor will have an incentive to sell, to prevent a portion of subsequent income from being allocated to this portion of the holding period. In either case, the investor could guess wrong: for example, an abnormally high return could be followed by an even higher return. Statistically, however, the investor will be better off selling the losers and keeping the winners. This tax-motivated behavior is not necessarily a fatal objection; after all, similar and even more powerful incentives exist under current law. The potential for this behavior, however, is a serious shortcoming in a system that purports to make the timing of sales a matter of indifference in tax planning.

These distortions arise under an interest charge approach that uses a formula, rather than annual valuations, to determine each year's deferred tax liability. One could imagine an interest charge approach that required such valuations, accepting the administrative burden that they would create. Such an approach would differ from a pure accre-

tion system only in allowing the *payment* of the tax to be deferred, with interest; the *amount* of the tax would be determined annually on a mark-to-market basis. Unlike the formula version, this mark-to-market version would not cause returns in any year to be "averaged out" among other years in the holding period.

Even this more refined interest charge approach distorts the investor's prospects for risk and reward. Consider a two-year investment, which goes up in value in the first year and down in value the same amount in the second. At the end of the second year, the asset is sold at an amount equal to its original purchase price. Under current law, the investor would recognize no gain or loss. Under the interest charge approach with annual valuations, however, the investor would owe interest on the deferred tax liability from the first year. While the tax liability itself might be sheltered by the loss sustained in the second year, the interest charge would remain.

This example is a special case of a more general consequence of the interest charge approach: a fixed charge for the use of the government's money in effect "leverages up" the investment. In financial parlance, leveraging refers to the greater potential for gain or loss that is present whenever an investment is financed in part with borrowed funds: if the gain on the investment is greater than the borrowing cost, the investor earns an extra profit based on the return on "other people's money." By contrast, if the investment earns less than the borrowing cost, or even loses money, the investor risks a greater loss. Imagine an investment that scores big in the first year, generating a large deferred tax liability. The investor is now under pressure in the second year to earn enough to cover the interest on the deferred tax, a pressure that another investor, buying the same investment for the first time in the second year, would not face.

An investor that was reluctant to accept the added risk arising from the interest charge would have the option of selling the asset and paying the tax, and might be given an option to pay the tax annually on a mark-to-market basis without actually having to sell the asset.

The first option assumes the asset itself can be readily sold; the second requires the investor to have other sources of liquidity. While this liquidity problem would vary greatly among taxpayers, it cannot be lightly dismissed. After all, once the deferred tax liability is determined on a mark-to-market basis, relief from the liquidity problem is the *only* advantage of the interest charge approach over a pure accretion system.

III. THE YIELD-BASED METHOD

There is a way to offset the benefits from tax deferral without the drawbacks of the interest charge approach. The key is to focus on the *yield* earned by the investment. This discussion in this Part assumes an investment that generates no interim cash flows, only a single contingent payment upon sale or maturity, such as a growth stock that does not pay dividends. A constant tax rate is also assumed. If there were no benefits from tax deferral, investments with the same pre-tax yields would have the same after-tax yields. In each case, the after-tax yield would be equal to the pre-tax yield times 100 percent minus the tax rate. For example, if the pre-tax yield is 10 percent and the tax rate is 35 percent, the after-tax yield would be 6.5 percent.

This focus on yield points the way to what the tax on sale should be. The government should take away just enough so that what the taxpayer has left is precisely what the taxpayer would have had if the return on the investment had been equal to the after-tax yield. If the taxpayer invests $1,000 at a 10 percent pre-tax yield, the investment will grow to $2,718 in ten years. If the investment were sold then, the tax should be an amount that would leave the taxpayer with $1,916, which is the proceeds of $1,000 invested for ten years at 6.5 percent. The difference of $802 is the proper amount of tax to compensate for the effects of deferral. This tax should be compared with the tax of $601 that would be imposed under the current system, which is obtained by applying the 35-percent tax rate to the nominal gain of $1,718. The extra $201 is what compensates for the privilege of deferring the tax until sale.

A. *An Aside on Compounding*

A reader who tries to verify the numbers in the preceding section with a pocket calculator will probably get different answers. The

difference has to do with the compounding of the investment return over time. If annual compounding is used, an investment of $1,000 at a 10-percent yield will grow to $2,594,[109] rather than $2,718 as stated above. The $2,718 figure is the result of continuous rather than annual compounding.[110] Although the choice of compounding methodology sounds like a technicality, it must be dealt with properly in order for any discussion of the effects of tax deferral to make sense.

Economists routinely use continuous compounding; the business community rejects it in favor of compounding over discrete intervals such as days, quarters, or years. The business practice makes sense because financial instruments pay off at discrete times, rather than dribbling out a continuous return. Moreover, daily compounding provides as close an approximation to continuous compounding as anyone could want. For example, with daily compounding the $1,000 investment with a 10-percent yield will grow to $2,717.91[111] over ten years, compared with $2,718.28 under continuous compounding (I had to report cents in order to show the difference). The additional 37 cents is all that is gained by using continuous rather than daily compounding.

Although daily compounding is a good approximation, economists use continuous compounding is because it is much easier to deal with mathematically. Also, continuous compounding reflects the continuous way that income accrues over time. Consider a $1,000 bond that pays $100 interest each year. While the bond yields a 10 percent annual return on the principal, the interest that accrues over the course of a year earns nothing until it is paid. The real return on the investment is therefore a blend of the 10-percent return on principal and the zero-percent return on the interest for the portion of a year

109 $2,594 ≈ $1,000 x $(1.1)^{10}$.

110 $2,718 ≈ $1,000 x $e^{(10\% \times 10)}$

111 $2,717.91 ≈ $1,000 x $(1 + (0.1/365)^{(10 \times 365)}$

that elapses between each instant that a bit of the interest is earned and the end of the year when the interest is paid. This blended return is about 9.53 percent.[112]

Because income is earned continuously, a tax system that allowed no tax deferral would provide tax assessment and payment on a continuous basis as well. Every nanosecond that income is earned, Uncle Sam would be there to claim his share.[113] While true continuous assessment is an impossibility, it is not hard to imagine daily assessment for items like bank account interest that are credited daily: the bank would simply withhold tax on a daily basis. Weekly wage earners suffer something close to continuous assessment, because of wage withholding.[114] Whatever the practical merits of more frequent assessment, tax deferral created by the realization requirement needs to be measured in a context of continuous assessment in order to separate out this deferral from the deferral that results from periodic assessment.

B. *Derivation of the Formula*

This section shows how to derive the formula for computing the tax under the proposed yield-based form of retrospective taxation. The mathematics is kept to a minimum, but some is unavoidable. The non-mathematically inclined reader can skim this section quickly.

When continuous compounding is used, the sales price of an investment can be expressed by inflating the purchase price by an exponential growth rate equal to its yield over the holding period:

(1) $$S_p = Pe^{jn},$$

112 This result is obtained using natural logarithms: ln 1.10 ≈ 0.0953.

113 *See* Strnad, Periodicity, *supra* note 15, at 1825–39.

114 Although wage withholding, which put the labor force under nearly continuous assessment, is now a humdrum fact of life, it took the crisis of World War II to put the nation in a political frame of mind to accept it. *See* Current Tax Payment Act of 1943, Pub. L. 78-68, § 1622, 57 Stat. 128–37.

where S_p is the pre-tax sales proceeds, P is the purchase price, i is the pre-tax yield in annual terms, and n is the length of the holding period in years or fractions thereof. In this formula, i and n could just as easily be expressed in terms of any time interval other than years, provided that they are expressed consistently, because their product does not depend on the length of the measuring interval. For example, a 10% annual yield over 10 years is exactly equivalent to a 2.5% quarterly yield over 40 quarterly intervals. Of particular interest is the pre-tax *holding period yield*, which is the yield expressed not in terms of any fixed calendar interval, but over the entire holding period itself. This pre-tax holding period yield p is given by the formula:

$$p = in. \tag{2}$$

Let the pre-tax *yield ratio* R_p be equal to S_p/P, the ratio of the pre-tax sales proceeds to the purchase price. Then, from Equations (1) and (2),

$$R_p = e^p. \tag{3}$$

Solving equation (3) for the pre-tax holding period yield p gives:

$$p = \ln R_p. \tag{4}$$

The holding period yield is the natural logarithm of the yield ratio. If the sales proceeds are less than the purchase price, the yield ratio will be less than 1, and the logarithm will be negative.

The after-tax holding period yield a is obtained by multiplying the pre-tax yield by 100% minus the tax rate t:

$$a = (1 - t)p = (1 - t)(\ln R_p). \tag{5}$$

Let S_a be what the sales proceeds would have been if the investment had earned the after-tax yield a rather than the pre-tax yield p. Similarly, let the after-tax yield ratio R_a be set equal to S_a/P, the ratio of the hypothetical sales proceeds S_a to the purchase price:

$$S_a = Pe^a; \quad R_a = e^a. \tag{6}$$

Substituting the value for a obtained in equation (5) for a in the second half of equation (6) provides a formula for the after-tax yield ratio in terms of the pre-tax yield ratio:

$$R_a = \exp\left[(1-t)(\ln R_p)\right] = R_p^{(1-t)}, \tag{7}$$

where $\exp(x)$ is an alternate notation for e^x. Conveniently, at this point natural logarithms and references to e drop out of the formula, and do not need to be dealt with again. The after-tax yield ratio is simply the pre-tax yield ratio raised to a power equal to one minus the tax rate. (This formula plays a role analogous to the principle under conventional tax law that after-tax gain is equal to pre-tax gain multiplied by one minus the tax rate. But the yield-based method works with yield ratios rather than directly with the amount of gain.)

The essence of the proposed method is that the hypothetical sales proceeds should be the amount the taxpayer has left after paying the tax. The tax T is simply S_p minus S_a, so that the taxpayer receives S_p, pays tax of T, and is left with S_a:

$$T = S_p - S_a = S_p - PR_a = S_p - PR_p^{(1-t)}. \tag{8}$$

The tax is the difference between the proceeds and the purchase price inflated by the after-tax yield ratio. The tax can also be stated solely in terms of the purchase price, the pre-tax proceeds, and the nominal tax rate:

$$T = S_p(1 - (S_p/P)^{-t}). \tag{9}$$

The yield-based tax has a confiscatory air at high yields. If an investment grows from $1,000 to $1 million, the yield-based tax is $910,875, which represents a nominal rate of tax of 91 percent on the $999,999 of gain. As the yield ratio gets larger, the nominal rate of tax approaches 100 percent.[115] Yet at these very high yields, even after a

[115] The nominal rate of tax is the tax divided by the gain $S_p - P$. Using the formula for the tax in Equation (9), this nominal rate works out to the following:

high nominal rate of tax, the after-tax amount that remains also grows, so that the proper after-tax yield is obtained. Unlike conventional interest charge approaches,[116] the nominal rate of tax can never exceed 100 percent.[117]

C. *Holding Period Neutrality*

Equation (9) shows that the tax does not depend at all on the holding period *n*. This result is in sharp contrast to interest charge approaches, where the interest charge increases as the holding period grows longer. This is not to say that the holding period is irrelevant. Most investments grow in value over time, and the greater the growth in value, the greater the pre-tax sales proceeds, which does of course factor into the formula. The key point is that the passage of time translates into a higher tax liability only to the extent that the investment makes more money.

The tax formula given in Equation (9) can be used to verify the tax computation offered in the beginning of this Part, for an investment that rose in value from $1,000 to $2,718, and a 35-percent tax rate:

(10) $$T = \$2{,}718 \times (1 - (\$2{,}718/\$1{,}000)^{-0.35}) \approx \$802.$$

Although in the example the investment was assumed to have been held for ten years, the tax computation would have been identical if the investment had earned the same return over five years.

$$\frac{1-(P/S_p)^t}{1-(P/S_p)}$$

As the yield ratio S_p/P becomes infinitely large, its inverse P/S_p approaches zero, and both the numerator and denominator of this fraction approach 1.

116 *See supra* Part II.B.3 (p. 303).

117 *But see* Gergen, *supra* note 5, at 234–35 (arguing that the effective rate of tax under the yield-based approach can exceed 100 percent).

Indeed, the tax would be the same if the investment had earned this return over ten *days*. What's going on?

It seems at first surprising that a tax that is intended to offset the benefits of deferral can be so oblivious to how long the deferral lasts. Yet under the yield-based approach the value of deferral is what the taxpayer makes of it. In the ten-year case, the taxpayer has earned a respectable 10-percent return, not only on the taxpayer's own money *but also on the government's money that was made available through tax deferral.* In the five-year case, the taxpayer earned a 20-percent return over five years, again with both the taxpayer's and the government's money. Even though the deferral period was only half as long as in the first case, the rate of return was twice as great, so the two effects cancel each other out. In the ten-day case, the deferral period is extremely short, but the rate of return is truly stupendous: 3,650 percent, when stated on an annual basis. Such an astronomical rate is not sustainable, but is occasionally realized over short periods in highly speculative investments such as call options.

This yield-based approach can be seen as a variant of the interest charge approach, where the interest rate charged is equal to the rate earned on the investment itself. While there is some truth to this view, it can be misleading. As discussed earlier, the difficulties with the interest charge approaches had to do with figuring out the principal on the deemed loan and the appropriate interest rate. If the interest rate is to be the rate of return on the investment itself, it would appear that periodic valuations would be necessary to determine both the principal amount and the interest rate.[118] Indeed, if a continuous compounding approach is used, the investment's value would apparently need to be determined every instant! Of course, as the formula

[118] At least one commentator has rejected the yield-based approach for just this reason. *See* Warren, *supra* note 5, at 478. A separate, valid, concern with the yield-based approach is that the purchase price may not be well defined in cases where the investment was not acquired for cash. *See* Blum, *supra* note 5, at 14, and *infra* Part V.A.3 (p. 361).

for the tax makes clear, no such determinations are necessary: the tax depends solely on the purchase and sales prices.

D. *Effect on Trading Decisions*

An ideal income tax would have no effect on the choice of investments or on decisions when to buy or sell. The realization requirement causes the current income tax to fall well short of that ideal. Investments that offer tax deferral are favored over investments that do not, and tax considerations favor the retention of appreciated investments and the sale of depreciated investments.

The yield-based method of retrospective taxation eliminates these biases. The formula was derived so that the after-tax yield on each investment is exactly proportional to its pre-tax yield. To the extent that investment decisions are based on expected yield, the yield-based method does not disturb the relative attractiveness of alternative investments. Volatility of yield is also a factor: investors tend to prefer safer investments over riskier ones. The yield-based method also preserves relative volatility: if one investment has a more volatile pre-tax yield than another, it will also have a more volatile after-tax yield.

In general, the yield-based method has an overall effect of reducing risk, because the government is in effect a partner, sharing in profits and losses. This sharing may cause some investors to become more tolerant of risk, and a safer investment that might have been chosen in the absence of tax considerations might be rejected in favor of a riskier one. This bias, however, is endemic to any form of taxing net income, and indeed might be considered a benign way to promote riskier ventures that might otherwise have difficulty obtaining capital.[119]

[119] It has even been argued that under an ideal income tax the higher return associated with the higher risk investments that would be acquired (because the tax law compresses the risk) enables investors to achieve on average the same after-

The yield-based method is "path independent" in that the amount of the tax does not depend on the path of the asset's fair market value over the holding period. The appreciation could have occurred early or late; the tax is the same. A consequence of this property is that the tax is immune to "strategic trading": the taxpayer cannot improve his or her position by a wash sale, in which the asset is sold and the after-tax proceeds are invested in an identical asset.

Consider the investment, discussed above, that grows from $1,000 to $2,718 over ten years. The tax is $802, reducing the yield from 10 percent to 6.5 percent and leaving the taxpayer with $1,916. Now suppose the taxpayer is considering a wash sale of the asset for $1,500 at the end of five years. A $1,500 value at the end of five years represents an annual pre-tax return of 8.11 percent with continuous compounding. Assuming a 35 percent tax rate, the corresponding after-tax return is 5.27 percent. If the original $1,000 investment had earned 5.27 percent over five years, the proceeds would be $1,302. So the tax imposed on the sale is $1,500 minus $1,302, or $198.

The after-tax proceeds of $1,302 are then reinvested in a lesser quantity of the original asset, and earn the same yield as the original asset over the last five years. The original asset increased in value from $1,500 to $2,718; applying the same rate of increase in value to the after-tax proceeds of $1,302 results in pre-tax proceeds of $2,359 on the ultimate sale of the reinvested asset. This represents a pre-tax yield of 11.89 percent over the second five years. At the corresponding after-tax yield of 7.73 percent, the proceeds would be $1,916, and the

tax return on the yield in excess of the risk-free rate that they would receive in a world without taxes. Joseph Bankman, *Commentary,* 50 TAX L. REV. 787, 790 (1995); Joseph Bankman & Thomas Griffith, *Is the Debate Between an Income Tax and a Consumption Tax a Debate About Risk? Does it Matter?,* 47 TAX L. REV. 377 (1992); David F. Bradford, Consumption Taxes: Some Fundamental Transition Issues 10–13 (NBER Working Paper No. 5290, 1995); Noël B. Cunningham, *The Taxation of Income and the Choice of Tax Base*, 52 TAX L. REV. 17 (1996); Evsey D. Domar & Richard A. Musgrave, *Proportional Income Taxation and Risk-Taking,* 58 Q.J. ECON. 388, 390 (1944); Alvin C. Warren, Jr., *How Much Capital Income Taxed Under an Income Tax Is Exempt Under a Cash Flow Tax?,* 52 TAX L. REV. 1 (1996).

tax would be $2,359 minus $1,916, or $443. Note that the after-tax proceeds of $1,916 are identical to the after-tax proceeds realized in the original case with no wash sale.

A similar result follows even if the wash sale results in a loss. Suppose the investment were worth $500 rather than $1,500 at the end of the first five years. In this case, the yield-based method provides a tax refund of $137, which reduces the negative yield over this period from -13.86 percent to -9.01 percent. Assuming the tax refund is made instantaneously available from the government, the taxpayer is in a position to reinvest in a greater quantity of the same asset than the amount sold: the pre-tax proceeds of $500 plus the tax refund of $137 can be reinvested in $637 in value of the asset. The original asset grew in value from $500 to $2,718 over the second five years, so the $637 reinvestment would grow to $3,464. The tax on the final sale is $1,548, reducing the yield over the second five years from 33.86 percent to 22.01 percent. When this tax is subtracted from the proceeds of $3,464, the taxpayer is left with $1,916, exactly as in the previous two examples. The taxpayer gets to keep $1,916 no matter what.

The immunity to strategic trading that is suggested by this example is proven generally in Part 1 of the Appendix. It can also be shown that the yield-based method is the *only* way of computing a retrospective income tax that is immune to strategic trading.[120] Any

[120] The condition of path independence can be expressed as a differential equation, since the increase (or decrease) in tax liability T from one moment to the next must be equal to the tax on the instantaneous return from the asset plus after-tax rate of return applied to the accrued tax liability at that time:

$$\partial T/\partial x = r_x t S_x + r_x(1-t)T,$$

where r_x is the asset's rate of return at moment x, and S_x is its value at that time. An income tax that satisfies this differential equation can be shown to have the formula structure of the yield-based method. A proof is given in Alan J. Auerbach, *Retrospective Capital Gains Taxation*, 81 AM. ECON. REV. 167, 172 (1991). Auerbach's own scheme of retrospective taxation is discussed *infra* in Part III.F.3 (p. 331).

interest-charge approach may reduce the benefits of strategic trading in particular instances, but cannot eliminate them in all cases.

E. *Path Dependence with Discrete Compounding*

The yield-based method is path-independent because the value of tax deferral is path independent. This view of tax deferral at first appears contrary to common sense; indeed, it is easy to construct a plausible-looking counter-example. Yet this path independence is provable mathematically, as shown in the Appendix, assuming that (i) continuous compounding is used, and (ii) the tax deferred is invested at the same return as the return on the underlying asset.

The second assumption is plausible in the case of an appreciating asset, because the taxpayer, by deferring the tax, is avoiding the need to sell off pieces of the asset over time to pay the taxes that would be continuously assessed under an accretion system. The assumption is less plausible in the case of an asset that declines in value, because the tax refunds that would be continuously provided under an accretion system might be invested in something else. Even in the case of an appreciating asset, the taxpayer might have paid the tax under an accretion system by borrowing, or by selling some other asset. For this and other reasons,[121] the yield-based method makes better sense when applied to a taxpayer's entire portfolio, rather than separately on an asset-by-asset basis.

Even when the tax deferred on a particular asset is assumed to be invested at the same return as the return on that asset, complications arise if discrete compounding intervals are used. To take a very simple case, suppose that an asset is bought for $1,000 at the beginning of year 1 and sold for $2,000 at the end of year 10, and annual compounding is used. In the first scenario, all of the appreciation occurs during year 1, so the asset has a value of 2,000 at the end of year 1.

[121] *See infra* Part IV.C.2 (p. 348).

The tax of $350 that would be due at the end of year 1 under an accretion system with a 35% rate is deferred until the end of year 10, but the taxpayer realizes no benefit from this deferral, because the asset itself does not appreciate during years 2 through 10. Thus, if the taxpayer had sold off a fraction of the asset worth $350 at the end of year 1 to pay the tax that would have been due under an accretion system, the taxpayer would have been in exactly the same position at the end of year 10 as it would be paying the $350 at that time under the realization-based system of current law.

In the second scenario, the value of the asset stays flat in years 1 through 9, and all of the appreciation occurs during year 10. Here no tax would have been due at the end of years 1 through 9 under an accretion system, because there was no appreciation in those years. Thus, there is no tax benefit from tax deferral under this scenario either! The scenarios of extreme front-loading and extreme back-loading of appreciation produce the same result.

Now consider a third, intermediate scenario, in which the asset appreciates at a constant percentage rate from one year to the next. A doubling in asset value over ten years represents an annual growth rate of approximately 7.2 percent annually. At the end of year 1, the asset would be worth $1,072, and a tax of $25 would be due under an accretion system. This $25 of deferred tax is assumed to be invested at a 7.2 percent return over the remaining nine years, as is the additional deferred tax on the asset that accrues at the ends of years 2 through 9. Under an accretion system with annual assessment, the taxpayer would have $1,580 by the end of year 10 (an after-tax return of 35 percent of 7.2 percent, compounded annually over ten years), but would have $1,650 under a realization-based system (35 percent tax applied to $1,000 of gain). Thus, where the gain is spread out over time, deferral is worth $1,650 minus $1,580, or $70.

It is striking that tax deferral should have no value in cases of extreme front-loaded or back-loaded appreciation, but should have significant value in the intermediate case where the appreciation oc-

curs more gradually. These effects are the consequence of using discrete rather than continuous compounding. Gain that accrues within a compounding period produces no tax deferral benefit during that period. The deferral benefit only arises to the extent of further appreciation during subsequent compounding periods. If the compounding period is short, and fluctuations in value are relatively smooth, the amount of gain or loss that accrues within any particular period is very small relative to the overall gain or loss. Consequently, compounding based on short periods, such as daily compounding, provides a very good approximation to continuous compounding, and the bizarre effects noted above would not arise to any significant degree. With compounding based on a longer period, such as annual compounding, the gain or loss that accrues within a particular period can amount to a substantial portion, or even all, of the overall gain or loss. In these cases, the benefits of tax deferral would be realized principally *within* the period in which the gain accrues, yet the effect of discrete compounding is to ignore these benefits.

For these reasons the choice of a compounding method is more than a matter of convention. Continuous compounding, although easiest to deal with analytically, is not necessarily the most realistic, because financial returns are typically paid in discrete intervals of no shorter than one day. Deferring tax from the morning to the afternoon of the same day is of no benefit if the shortest investment maturities are at least overnight. The most realistic choice is likely to be daily compounding, which very closely approximates continuous compounding in all but the most unusual cases. The formulas presented here are nonetheless based on continuous compounding, in order to keep the mathematics as simple as possible.

F. *Philosophical Underpinning*

1. *Irrelevance of Periodic Valuations*

Path independence liberates the yield-based method from any concern over fair market value other than when the asset is bought and when it is sold. In contrast to accretion and interest-charge systems, the yield-based method requires neither periodic valuations nor accounting conventions that substitute for valuations. By making intermediate values irrelevant, the yield-based system preserves an aspect of the realization requirement that is very much worth preserving.

Unrealized gains are in some sense unreal. Academics scoff at this idea, noting that a mere sale, itself not a wealth-creating event, can turn unrealized gains into very real cash.[122] Yet since the taxpayer has decided not to sell, the unrealized gain is based on a contrary to fact hypothesis. If the owner of a closely held business realizes x upon a sale of the business at time t_1, the fact that she could have realized y upon a sale at some earlier time t_2 is of only academic interest to her, since in fact she did not sell at that time. The same is true even if the asset is not a closely held business but is a portfolio of publicly traded securities, and the amount obtainable at time t_2 can be ascertained with a fair degree of accuracy. Of course, the fact that investors regard unrealized gains as "paper profits" is no justification for giving investors the deferral benefits offered by the realization requirement. But a tax formula that eliminates this deferral benefit without relying on interim valuations is a promising way out of this dilemma.

As shown below, the yield-based method is forced to rely on interim valuations when applied to investments with multiple cash flows. Even in this context, however, the stakes can be kept fairly low,

[122] *See, e.e.,* Blum, *supra* note 12, at 248–50.

so there is no need for strenuous efforts to determine precise valuations.

2. *The Government as an Equity Partner*

The imposition of an income tax makes the government a silent partner in every business venture. The government shares in your profits and, subject to limitations, it shares in your losses as well. The fact that an investment was made with an expectation of profit is itself no reason for imposing a tax if the profit never materializes.[123]

Recent inroads against the realization requirement threaten this principle. Contingent payment obligations are taxed based on a projected schedule of payments that presumes a positive investment return,[124] even though the contingent nature of the obligation makes the realization of such a return uncertain. The overtaxation that occurs in these cases is primarily a matter of timing, because an offsetting loss deduction will usually be allowable if the actual returns are lower than projected. Yet this timing detriment deprives the investor of the return on the funds used to pay the tax, and so the investor suffers a tax cost when the investment fails to make money even after the ultimate loss deduction is taken into account.

Interest-charge approaches present a similar problem. They assume that the taxpayer will earn a return on the deferred tax at a rate at least equal to the rate on which the interest charge is based. When the interest charge is based on prevailing risk-free rates, it may seem plausible to assume that the investor can earn such a return. If the investor chooses to invest in something riskier, at least the expected value of this return will be at least equal to the risk-free rate, or the investment would not be made. Yet a risky investment with a high expected return can turn out badly. The tax on such an investment is

123 *See infra* Part III.F.3 (p. 331).

124 *See* Gergen, *supra* note 5, at 219–21.

based on the actual return, not the expected return. But if the investment is financed with deferred taxes from some other investment, an interest charge approach would impose a tax cost in any case where the investment failed to earn the interest-charge rate.

For example, suppose investment *A* has appreciated (or is deemed, under some accounting convention, to have appreciated) by $1,000 in the first year, and the accrued tax liability of $350 is deferred but is made subject to a 6.5-percent nondeductible interest charge (which would be equivalent to a 10-percent deductible charge). The interest charge is fixed at $23 (6.5 percent of $350) regardless of how the taxpayer's investments fare in the following year. If the taxpayer only breaks even, and sells the investment for the same price at the end of the second year, the $23 will be due even though the taxpayer derived no benefit from the deferral. Such an interest charge represents taxation based on expected rather than actual returns.

The yield-based method eliminates this problem by treating the deferred tax as an equity investment by the government rather than as an interest-free or interest-bearing loan. The government becomes a silent partner in the reinvestment of the deferred tax. The government maintains its investment as long as the taxpayer does, and when the taxpayer cashes out, the government does, too. The government charges for tax deferral only if the taxpayer actually benefits from it.

This feature of the yield-based method reduces investment risk, and links taxes more closely to ability to pay. By contrast, interest-charge approaches increase risk, because the interest charge introduces an element of investment leverage that amplifies above-average returns, but penalizes below-average returns. This leverage is reduced but not eliminated by interest-charge methods that use intermediate valuations that are based on formulas subject to retrospective adjustment. For example, the interest charge for PFICs is based on an assumption of straight-line appreciation over the holding period, but the slope of the straight line is not determined until a sale or extraor-

dinary distribution occurs.[125] Even in such a case, however, the taxpayer is overcharged for tax deferral if the investment earns less than the PFIC interest charge, which is three points above Treasury rates.

3. *Comparison with Ex Ante Approaches*

The yield-based method is based on *ex post* results rather than *ex ante* expectations. Yet if the goal is simply to deter behavior that is motivated by the realization requirement, such as the lock-in effect, a tax based on *ex ante* expectations can do the job. Alan Auerbach has designed such a method of retrospective taxation.[126] In its formal structure, Professor Auerbach's approach is identical to the yield-based method, but his method charges interest on the tax deferred at the risk-free rate, and also treats each asset as if it had appreciated at this risk-free rate.[127] Under both methods, the rate of return on the asset is the same rate used to offset the benefits of deferral, but since his tax formula assumes that all assets appreciate at the risk-free rate, the resulting tax is not an income tax at all, and even sales of assets that have in fact gone down in value attract tax liability. Professor Auerbach acknowledges this characteristic, but suggests that the overall administrability of his scheme outweighs the unfairness that can arise in individual cases.[128]

Professor Auerbach's method has been generalized by David Bradford to take into account actual investment performance.[129] Professor Bradford's method, like Professor Auerbach's, taxes each investment as if it had accrued in value at the risk-free rate. In addition, the difference between the sales proceeds and the basis (adjusted

125 *See supra* notes 88–90 and accompanying text.

126 Auerbach, *supra* note 120.

127 *Id.* at 169–73.

128 *Id.* at 176.

129 David F. Bradford, *Fixing Realization Accounting: Symmetry, Consistency and Correctness in the Taxation of Financial Instruments,* 50 TAX L. REV. 731 (1995).

to reflect accreted income at the risk-free rate) is treated as further gain or loss that is also taken into account.[130] What makes Professor Bradford's method immune to strategic trading is that the gain or loss is computed as of a date (the "reference date") that is established at the time the investment is acquired.[131] Since the actual sales date is unlikely to be the same as the pre-established reference date, the gain or loss is taken into account at its present value as of the reference date, using the risk-free rate as the discount rate. The tax is paid on the sales date, with an interest charge on the tax deferred from the reference date to the sales date. If the sales date precedes the reference date, the investor is entitled to an interest credit on the "prepaid" tax.[132]

In effect, the reference date is an arbitrary date on which any variation in return from the risk-free rate is deemed to have instantaneously accrued. It is intended to be completely arbitrary, and could even be made elective with the taxpayer, provided it is set in advance. Even the tax *rate* that is applied to this variation from the risk-free return could be set to any arbitrary value, and Professor Auerbach's method can be seen as a special case where this rate is zero.[133] Because any return in excess of the risk-free rate is deemed to have accrued on the reference date, and interest is charged on the tax deferred from that date, the interest charge will be excessive if the actual gain in fact accrued later.

These *ex ante* methods are immune to strategic trading in the sense that an investor cannot improve her *expected* position by making a wash sale and paying tax (or receiving a refund) at that time. Any system which, like those of Professors Auerbach and Bradford, charges interest on deferred tax at the risk-free rate, will allow inves-

130 *Id.* at 772.

131 *Id.* at 770.

132 *Id.* at 770–72.

133 *Id.* at 776–77.

tors to obtain a benefit from deferral if they in fact reinvest the deferred taxes at a higher rate. But no investor can be assured of ob-obtaining such a higher rate.

The yield-based method seeks a stronger form of immunity from strategic trading, in that the investor's *actual* position cannot be improved by making a wash sale, regardless of whether actual invest-investment performance follows expectations. The goal of the yield-based method is to put the investor in the same position on the maturity date of the investment as would be obtained under an accretion system, regardless of the amount and timing of the investment's return. The advantage of an *ex post* approach is that the investor is taxed on what actually happened, not on what might have happened. The fact that an investor could have invested deferred taxes at the risk-free rate is no reason to impose a charge at this rate if in fact the taxpayer invested these deferred taxes at a different rate.[134]

Under an *ex post* approach, the actual return earned on deferred taxes takes center stage. In the case of a single asset with a single payoff at maturity at a gain, it seems clear that the deferred taxes have been invested in the asset itself. Complications arise, however, with investments that generate multiple cash flows, investments sold at a loss, and portfolio effects. It is to these complications that we now turn.

G. *Investments with Multiple Cash Flows*

The yield-based method seems almost too good to be true, and in a sense it is. The basic case discussed so far involves a single investment with a single payoff at maturity. Real investments typically provide intermediate as well as final payments, and sometimes additional investments in an asset must be made subsequent to the initial

[134] Otherwise, the tax on capital would resemble an "underachiever's tax," under which earned income is taxed based on what people *could* earn, rather than on what they *do* earn.

purchase. These more realistic types of investments present serious conceptual difficulties as well as more complicated calculations, all of which raise formidable obstacles to any practical implementation of the yield-based approach. Yet an exploration of these obstacles, and potential ways around them, teaches much about the economics of tax deferral.

1. *The Problem of Interim Payments*

Imagine an investment of $1,000 with two uncertain payoffs at times t_1 and t_2. At time t_1 $500 is received. What tax should be imposed, if any?

Our conceptual thinking has long since passed the stage of trying to answer this type of question by looking for formal indicia of whether the $500 payment represents a return on, or a recovery of, the taxpayer's investment, although the tax law has not always kept up. If the investment were corporate stock, the $500 would be taxable to the extent of the corporation's earnings and profits; the balance would be a tax-free recovery of capital.[135] If the investment were a contingent payment debt obligation, the payment would be taxable to the extent that it exceeds the amount assumed in drawing up the projected payment schedule prepared upon issuance to compute the accrual of original issue discount.[136]

The trouble is, there is no way of knowing at time t_1 whether the investment will make or lose money. Everything depends on the payoff at time t_2. So a "wait and see" approach might be warranted: treat the investment as an "open transaction"[137] until time t_2, and then use the yield-based method to compensate for the effects of tax deferral.

135 I.R.C. § 301(c).

136 Treas. Reg. § 1.1275-4(b).

137 *See* Burnet v. Logan, 283 U.S. 404 (1931). For an assessment of the current state of the "open transaction" doctrine, *see* Robert R. Wootton, *Mrs. Logan's Ghost: The Open Transaction Doctrine Today,* 71 TAXES 725 (1993).

In principle, such an application of the yield-based method could work. At time t_2, the overall yield on the investment will be known. One could compute what the final payment at maturity would be if the yield were scaled back to the appropriate after-tax yield, and the tax would be the difference between the hypothetical payment at maturity and the actual payment.

A practical difficulty is that the hypothetical payment, so computed, might be negative. In such a case, the taxpayer will have already received enough from the interim payment to achieve the after-tax yield, and therefore the tax will exceed the amount of the final payment. This would obviously be true in the case of a $1,000 investment that yielded an interim payment of $2,000 and a final payment of only $10. If the government failed to collect any tax when the $2,000 payment was received, it would have to look beyond the final $10 payment for the cash to pay the tax on maturity. Such an outcome frustrates a principal objective of the realization requirement, which is to impose the tax when the cash comes in. The tax collector should not step in too early, but should not step in too late, either.

A more fundamental problem with this wait and see approach is that it assumes that the interim cash flows can be reinvested at the same yield to maturity as the underlying asset. Even if the underlying asset is of a mass fungible type, so the taxpayer can feasibly buy more of it, there is no assurance that the price of that asset will enable the taxpayer to achieve the same yield to maturity on the incremental investment.

Returning to the original example, suppose the investment that pays $500 at t_1 pays an additional $1,000 at t_2, and that t_1 and t_2 occur one and two years after the original investment, respectively. The pretax yield on this investment is 24.7% percent, using continuous compounding.[138] The corresponding after-tax yield, based on a 35% tax

[138] This rate of return is determined by solving the following for r:

$$\$1{,}000 = \$500e^{r} + \$1{,}000e^{-2r}.$$

rate, is 16.1 percent. This after-tax yield would be realized by adjusting the final payment from $1,000 to $792, so the tax on the final payment under the yield-based method would be $1,000 minus $792, or $208. By contrast, under current law the tax on the final payment, assuming the interim payment were treated as a return of capital, would be 35 percent of $500, or $175. The additional $33 of tax compensates for the deferral.

Imposing a tax on the final $1,000 payment is not the only way to scale back the yield. A tax could have been imposed on the interim $500 payment, and a smaller tax on the final payment. For example, if $100 of tax had been imposed on the interim payment, the tax on the final payment could have been reduced by $116, and the after-tax yield would have been the same. The equivalence of these cases, and of either case to a scheme of accretion taxation, depends on the investor's being able to reinvest the interim payment at the same pre-tax yield as the original investment.

Suppose the original investment were worth $1,000 immediately after the interim payment, so that the entire pre-tax yield accrued in the first year, and no additional appreciation occurred in the second year. A reinvestment of the interim payment in the same type of asset would produce a zero return in the second year. In such a case, the taxpayer would be better off paying tax on the interim payment, since each $1 of tax on the interim payment reduces the tax on the final payment by $1.16.

Conversely, if the investment were worth $700 after the interim payment, the increase in value during the second year to $1,000 represents a return of 35.7 percent. In this case, the taxpayer would prefer to avoid paying any tax on the interim payment, because each $1 reinvested at the end of year 1 yields $1.36 at the end of year 2, and the

This is a quadratic equation in e^r, and can be solved using the quadratic formula. In more complicated cases, the solution can be obtained by successive approximations using algorithms built into spreadsheet programs and pocket calculators.

additional return is more than enough to pay the additional tax on the final payment.

In short, the situation is hopelessly path dependent. Any attempt to correct for the benefits of tax deferral through retrospective taxation has to deal with the problem of reinvestments of interim payments, and the yields on these reinvestments will depend on fair market values at the time the interim payments are made. The market cannot be ignored at these times, because the investor is quite literally "in the market" to reinvest the interim payments.

2. *An Incentive for Accurate Valuation*

A theoretically correct tax could be computed on each interim payment from an investment with multiple cash flows, if the value of the investment were known on the date of the interim payment. The interim payment can be seen as a *divestiture* by the taxpayer of a proportionate part of the investment equal to the fraction of the investment's value that is represented by the interim payment.[139] If an investment is worth $1,000 just before making an interim payment of $100, then the payment represents a divestiture of 10% of the taxpayer's interest. A corresponding 10% of the taxpayer's cost can be attributed to this payment, which is then treated as the payoff from a separate single-payment investment and taxed under the yield-based formula.

For example, if an investment cost $1,000 and was worth $1,200 just before an interim payment of $400, then the interim payment represents a cashing out of one-third of the investment's value. One-third of the cost is $333, and the tax is:

(11) $$T = \$400 \times (1 - (\$400/\$333)^{-0.35}) \approx \$25.$$

[139] Professor Auerbach, for example, notes that except in their tax treatment, a dividend is equivalent to a partial share repurchase. Auerbach, *supra* note 120, at 175.

(Under current law the tax would be $23, which is 35% of the difference between $400 and $333.[140] The extra $2 compensates for tax deferral.) After the interim payment is received, the taxpayer's cost basis is reduced by the $333 that is applied against the interim payment, to $667. If the final payment is $1,000 an additional tax of $132 is due at that time:

(12) $$T = \$1{,}000 \times (1 - (\$1{,}000/\$667)^{-0.35}) \approx \$132.$$

These calculations depend on an assumed value of the investment at the time the interim payment is received. In any actual case, the value used to calculate the tax might be greater or less than the unknown (and possibly unknowable) "perfect" valuation that might be offered by an omniscient appraiser. If the value is on the low side, then the tax on the interim payment is too low, and the taxpayer has in effect borrowed the difference from the government. If the value is on the high side, then the tax is too high, and the taxpayer has in effect lent the difference to the government. In each case, the implicit rate of interest on the loan to or from the government is the after-tax rate assumed to be earned on the investment after the interim payment is received, based on the valuation used in calculating the tax.

An interesting set of incentives arises if the taxpayer is assumed to reinvest the interim payment (less the tax) in more of the same investment as that which generated the payment. The taxpayer's after-tax return on this reinvestment is equal to the return on the original investment over the balance of the holding period, which will be based on its actual value (*i.e.,* the value that would be identified by a "perfect" valuation) at the time of the interim payment. This actual value (rather than the assumed value used to calculate the interim tax) governs the rate of return on the reinvestment because the taxpayer

[140] This assumes that the interim payment is in the form of a sale or exchange transaction, such as a redemption of stock by a corporation. If so, under I.R.C. § 1012, the taxpayer would recover the pro rata share of basis.

must buy the investment at its actual value in order to reinvest the interim payment.

If the value used to calculate the tax on the interim payment is lower than the actual value, then the tax is based on an assumed rate of return on the investment that is understated for the period before the interim payment and is overstated for the period after. As noted above, however, this overstated rate is the rate at which the taxpayer is implicitly borrowing from the government by paying too little tax. This is a bad deal for the taxpayer, who is borrowing from the government at a higher rate than is earned on the reinvestment of the interim payment.

Things are just as bad if a higher value is used to calculate the tax on the interim payment. This tax is based on an assumed rate of return that is overstated before the interim payment and understated thereafter. The understated rate, however, is the rate at which the taxpayer is implicitly lending to the government by paying too much tax. The taxpayer could do better by paying less tax, and reinvesting the difference at the actual, rather than the understated, rate of return. As it turns out, *the taxpayer is best off if the tax on the interim payment is calculated based on an assumed value of the investment that is equal to its actual value.* Under these circumstances, the taxpayer can be trusted to do the valuation; there is no incentive to cheat. A proof is provided in Part 2 of the Appendix.

Suppose, in the example described above, the taxpayer is considering a range of values between $1,000 and $1,400 for the investment at the time of the interim payment. Table 1 illustrates the consequences of the choice of value. As the assumed interim value increases, the tax on the interim payment goes up, while the tax on the final payment goes down by an even greater amount, to offset the taxpayer's lost earnings from having to pay more tax early.

The most striking feature of this table is how little the taxpayer is affected by the choice of value. The top row shows a $400 range of values, yet the effect in the bottom row on the taxpayer's after-tax

proceeds when the investment matures is so small that the table had to display cents to show the differences fully. Within this narrow range of outcomes, the maximum proceeds to the investor of $1,301.55 arise when the tax is based on an assumed value of $1,200 at the time of the interim payment, which in this example is the actual value that a perfect valuation would provide.

TABLE 1

Investment Characteristics:	
Tax Rate	35%
Purchase Price	$1,000
Actual Interim Value	1,200
Interim Payment	400
Final Payment	1,000

Assumed Interim Value	$1,000.00	$1,100.00	$1,200.00	$1,300.00	$1,400.00
Tax on Interim Payment	0.00	13.12	24.73	35.09	44.44
Tax on Final Payment	163.72	146.32	132.30	120.77	111.10
After-Tax Final Proceeds	836.28	853.68	867.70	879.23	888.90
Amount Reinvested	400.00	386.88	375.27	364.91	355.56
Pre-tax Reinv. Proceeds	500.00	483.60	469.09	456.13	444.45
Tax on Reinvestment	37.56	36.33	35.24	34.27	33.39
After-Tax Reinv. Proceeds	462.44	447.26	433.85	421.86	411.06
Total After-Tax Proceeds	$1,298.72	$1,300.95	$1,301.55	$1,301.10	$1,299.97

IV. PORTFOLIO-BASED TAXATION

A. *The Loss Paradox*

When discussing an income tax, the examples of transactions that first come to mind are those that make money. Not all do. The flip side of a tax on gains is a tax refund for losses.[141] The yield-based formula can be applied in the same manner to losses as well as gains. An investment of $1,000 that is sold for $900 has a pre-tax return of negative 10.54 percent with continuous compounding. The investment will have an after-tax return of negative 6.85 percent (65 percent of 10.54 percent) under the yield-based formula, assuming a 35-percent tax rate, and the after-tax proceeds will be $933.81, which is the pre-tax proceeds of $900 plus a $33.81 tax refund.

Under a conventional realization-based tax, the refund would be 35 percent of the $100 loss, or $35. The yield-based formula provides a refund that is smaller by $1.19, to compensate for the effects of deferring the recognition of the loss until the realization event occurs.

The paradox is that the refund is smaller rather than bigger. After all, the taxpayer has deferred enjoyment of the tax benefit of the loss. If taxpayers who defer gains are subject to a bigger tax, it seems plausible that taxpayers who defer losses should get bigger refunds.

It doesn't work that way; indeed, the more dramatic the loss, the more the refund becomes relatively meager.

Table 2 shows the tax refund under a conventional tax and under the yield-based formula, as the proceeds of a $1,000 investment approach zero. The refunds under the yield-based formula not only fail to keep up with the refunds under current law, they actually start to drop after the losses begin to exceed 70 percent.

[141] The discussion here refers loosely to "refunds" as if every transaction resulted in a cash flow to or from the government. In practice, the so-called "refund" might be an offset to tax liability on other income, rather than a separate cash payment.

TABLE 2

Pre-Tax Proceeds	*Current-Law Refund*	*Yield-Based Refund*
$900	$35.00	$33.81
800	70.00	64.98
700	105.00	93.07
600	140.00	117.46
500	175.00	137.28
400	210.00	151.24
300	245.00	157.22
200	280.00	151.29
100	315.00	123.87

TABLE 3

Pre-Tax Proceeds	*Current-Law Refund*	*Yield-Based Refund*
$90	$318.50	$119.05
80	322.00	113.65
70	325.50	107.55
60	329.00	100.62
50	332.50	92.67
40	336.00	83.41
30	339.50	72.36
20	343.00	58.64
10	346.50	40.12
5	348.25	26.94
1	349.65	10.22
0.5	349.83	6.65
0.1	349.97	2.41
0.0	350.00	0.00

Table 2 is continued in Table 3 to show severe losses of substantially all of the original investment. In the limiting case, where the investment is lost completely, current law provides a refund of $350, and the yield-based formula provides nothing.

This is heads-I win, tails-you-lose with a vengeance. The yield-based formula adds to the tax on gain investments, but subtracts from

the refund for loss investments. Yet this result makes sense in the context of the underlying assumptions of the yield-based formula. For gain investments, the taxpayer is assumed to reinvest the tax deferred at the same yield as that earned by the underlying investment. This assumption is plausible because the taxpayer retained the investment throughout the holding period without having to liquidate any portion prematurely to pay tax on gains as they accrued.

For loss investments, the assumption is that the taxpayer would have invested any interim tax refunds at the same (negative) yield as that suffered by the underlying investment. For such an investor, the time value of money is negative. The normally detrimental effect of deferring the tax refund on interim losses is flipped around by the negative reinvestment rate.[142]

The assumption that a holder of a losing investment would have invested interim refunds in more of the same losing investment has the methodological pitfalls of any contrary-to-fact assumption. Who is to say what the investor would have done? Yet this assumption is essential to the workings of the yield-based formula, even as applied to gain investments. An investment that produces a net gain may have suffered periods of depreciation in value along the way. The yield-based tax is immune to strategic trading only if the same formula is used to compute the tax for losses as well as gains.

B. *The Problem of Non-Linear Taxation*

Difficulties with the yield-based method run deeper than an apparent pro-government bias in its treatment of losses. Even when applied to gain investments, the yield-based method is *non-linear*: the sum of the taxes on two separate investments is generally unequal to

[142] At least one commentator has considered and rejected the yield-based tax, in part for its treatment of losses. *See* Gergen, *supra* note 5, at 235 n.81.

the tax that would be imposed on the same two investments viewed as a single unit.

Consider two investments, each bought for $1,000, one of which is sold for $1,200, and the other of which is sold for $2,000 on the same date. An equivalent single investment would have cost $2,000, and would have been sold for $3,200. The tax under current law and under the yield-based method would be computed as follows:

TABLE 4

	First Investment	*Second Investment*	*Total*	*Equivalent Single Investment*
Current Law				
Cost	1,000	1,000		2,000
Proceeds	1,200	2,000		3,200
Gain	200	1,000		1,200
Tax	70	350	420	420
Yield-Based Method				
Pre-Tax Yield Ratio	1.2000	1.0000		1.6000
After-Tax Yield Ratio	1.1258	1.5692		1.3573
After-Tax Proceeds	1,126	1,569	2,695	2,715
Tax	74	431	505	485

Unlike current law, the yield-based method produces a total tax on the two investments ($505) which is greater than the tax on the same two investments viewed together ($485). This holds true as a general rule: the yield-based method always produces more favorable results for the taxpayer when investments are aggregated.

This effect can be seen even more sharply by looking at a pair of investments that achieve a perfect pre-tax hedge, and therefore should, under a rational scheme of taxation, achieve a perfect after-tax hedge. Consider the so-called "bull and bear" bonds; the principal amount of each is linked to a stock index, but the bull bond goes up when the stock index goes up, and the bear bond goes up when the

stock index goes down.[143] In effect, these are instruments that have embedded long and short positions, respectively, on the stock index.

A taxpayer that holds both a bull and a bear bond is hedged with respect to the index, and will realize a net return exactly equal to the interest payable on the bonds.[144] Disregarding the interest element of the return for simplicity, a taxpayer that holds both bonds will exactly break even, and therefore should owe no net tax. While current law provides this result in most cases,[145] the yield-based method produces a net positive tax liability in every case except the degenerate case in which the stock index fails to move at all. Suppose the investor owns a $1,000 bull bond and a $1,000 bear bond, and the index goes up 50 percent, causing the payoffs at maturity to be $1,500 for the bull bond and $500 for the bear bond. The yield-based tax is shown in Table 5, and generates a net tax of $61 on a combined investment that breaks even. This asymmetrical treatment of gains and losses turns the capital markets into a sucker's casino.

TABLE 5

	Bull Bond	*Bear Bond*	*Total*
Cost	1,000	1,000	2,000
Proceeds	1,500	500	2,000
Pre-Tax Yield Ratio	1.5000	0.5000	
After-Tax Yield Ratio	1.3015	0.6373	
After-Tax Proceeds	1,302	637	1,939
Tax	198	-137	61

143 *See* Jill Dutt, *Daiwa Unveils New Bull and Bear Bonds,* INVESTMENT DEALER'S DIG., June 16, 1986, at 16.

144 *See id.* at 16.

145 There is some potential under current law for a fully hedged taxpayer to incur a positive tax liability if the contingent bond regulations apply, because the gain on the bond that goes up in value is treated in full as ordinary income, but the loss on the other bond is treated as an ordinary loss only to the extent of realized interest income, and any remaining loss is a capital loss. Treas. Reg. § 1.1275-4(b)(6).

It is worth trying to understand why the yield-based method generates such unfavorable results based on seemingly neutral assumptions about reinvestment. These assumptions are that tax deferred on a gain investment is reinvested in that investment, and that tax refunds deferred on a loss investment would have been invested in that investment. A proper pre-tax hedge violates these assumptions. Consider, in the bull and bear bond case, a moment in time when the bull bond inches up and the bear bond inches down. Under an accretion system, these market movements would generate a tax liability on the bull bond and a tax refund on the bear bond. The yield-based approach assumes that the investor would sell a piece of the bull bond to pay the tax, and reinvest the tax refund in more of the bear bond. The reality is likely to be quite different: the investor would use the refund on the bear bond to pay the tax on the bull bond, while leaving the investments untouched, thereby maintaining the positions in properly hedged amounts.

Current law has its own problems with aggregation. These problems arise principally because of the disparities among the rules that govern the timing and character of income for different types of investments. Consider a taxpayer with a weak functional currency that acquires a zero-coupon debt instrument in a strong currency, with a correspondingly low implicit interest rate, and simultaneously sells forward the strong currency payment that is due on the instrument at maturity. Absent the integration rules of the foreign currency regulations, the taxpayer could accrue interest at the low strong currency interest rate, while locking in a net capital gain at maturity by reason of the forward sale contract. Instead, these integration rules require the two transactions to be treated as a single equivalent investment in an instrument denominated in the taxpayer's own functional curren-

cy.[146] Similar issues arise with swaps that contain embedded loans, and loans that contain embedded options.[147]

The yield-based method was supposed to avoid issues of bifurcation and integration, yet its bias in favor of integration is pervasive. As a result, meaningless formal differences can result in significant differences in tax liability. These considerations indicate that the yield-based method cannot be applied to investments in isolation from each other. A portfolio approach is needed.

C. *The Portfolio View*

1. *Managing Risk and Expected Return*

A good portfolio is more than the sum of its parts. Effective portfolio management is more than a set of investment decisions that, viewed individually, make good sense. In its essentials, portfolio management is based on two principles: first, riskier investments have higher expected returns; and second, proper diversification can reduce risk without reducing the expected return.

The benefits of diversification have been understood long enough to be embodied in the common law of fiduciary duties.[148] What is new is the mathematical techniques that are used to quantify the types of risk that can be reduced through diversification.[149] Armed with these

146 *See* Treas. Reg. § 1.988-5(a)(8)(iii).

147 *See* Frank V. Battle, Jr., *Bifurcation of Financial Instruments*, 69 TAXES 821, 823 (1991).

148 *See* AUSTIN WAKEMAN SCOTT & WILLIAM FRANKLIN FRATCHER, THE LAW OF TRUSTS § 230.3 (4th ed. 1988).

149 The modern era can be dated to the publication of the capital asset pricing model, which separates risks on corporate stocks into a market-correlated element, which is unavoidable, and a company-specific element, which can be eliminated through diversification. In short, the model concludes that expected equity returns will reflect only the first category of risk, since investors can eliminate the second category. *See* PETER L. BERNSTEIN, CAPITAL IDEAS 41–60 (1992).

techniques, portfolio managers can construct a portfolio to achieve any desired balance between risk and expected return.

As particular items in a portfolio change in value, an investor will need to sell some items and buy others in order to maintain the desired balance in the overall portfolio. Consider, for example, a portfolio that is invested 50 percent in a basket of stocks, and 50 percent in Treasury bills.[150] If the stocks double in value but the value of the Treasury bills stays the same, then two-thirds of the portfolio value will be represented by stocks. The investor will need to sell some of those stocks, and purchase more Treasury bills, to restore the initial balance.

Ideally, the tax law would not hinder this portfolio rebalancing. In reality, it does, because the realization requirement has a lock-in effect that discourages sales of appreciated assets. The yield-based approach to retrospective taxation eliminates this lock-in effect by eliminating the tax penalty for selling sooner rather than later. Moreover, a portfolio-wide view addresses the principal problems that arise under the yield-based method when applied to investments separately. First, the non-linear nature of the tax becomes less problematic, because all of the investments of a particular investor are viewed in the aggregate. Second, on a portfolio-wide view, deferred taxes are assumed to be reinvested at the portfolio's overall rate of return, rather than at the rate of return earned by the particular investments that generated the deferred taxes. This assumption is far more consistent with the behavior of a rational portfolio manager.

2. *Applying the Yield-Based Approach to a Portfolio*

When the yield-based approach is applied to a portfolio, the entire portfolio is treated as a single investment with multiple inflows and outflows. Each time an inflow or outflow occurs, the portfolio as a

[150] This is a perfectly legitimate way to reduce overall stock market risk. *See* Bernstein, *supra* note 149, at 72–73.

whole is valued on a pre-tax basis. At each such time, the portfolio is also valued on an after-tax basis, which is the amount of funds invested, adjusted by after-tax cash flows and after-tax yields for each accru-accrual period between valuation dates. The difference between the pre-tax value and the after-tax value at any point in time is the deferred tax liability, which presents the government's investment in the portfolio.

Any realization event is treated as a partial divestiture of the entire portfolio. A proportionate amount of the after-tax value of the portfolio is assigned to the amount realized, and the difference between the two is the tax liability at that time.

TABLE 6

	Initial Value	*Valuation Dates* 1	2	3	4
Pre-Tax Value	$0	$1,500	$2,700	$2,300	$1,000
Cash Received	0	0	100	300	500
Cash Invested	1,000	500	81	0	549
New Pre-Tax Value	$1,000	$2,000	$2,681	$2,000	$1,049
Pre-Tax Yield Ratio		1.5000	1.3500	0.8579	0.5000
After-Tax Yield Ratio		1.3015	1.2154	0.9052	0.6374
After-Tax Value		$1,302	$2,190	$1,982	$1,098
After-Tax Amount Received		0	81	259	549
Amount Invested		500	81	0	549
New After-Tax Value		$1,802	$2,190	$1,723	$1,098
Deferred Tax		$198	$510	$318	$ (98)
% Divested		0.00%	3.70%	13.04%	50.00%
Tax Due		0	19	41	(49)
Net Cash Flow	$-1,000	$-500	0	$259	0

To construct an example, we need a set of cash flows at arbitrary intervals separated by valuation dates, as shown in Table 6. The table is divided into four parts. The first part shows the portfolio value on a pre-tax basis on each valuation date, both before and after adjustments to reflect dispositions and fresh investments.

The second part shows the portfolio's yield ratio for the accrual period on a pre-tax and after-tax basis. The after-tax yield ratio is determined in the usual way, by raising the pre-tax yield ratio to a power equal to one minus the tax rate.

The third part of the table tracks the after-tax value of the portfolio. The after-tax value on each valuation date is determined by adjusting the after-tax value on the preceding valuation date (after taking into account cash flows on that preceding valuation date) by the after-tax yield ratio for the accrual period. This value is decreased by cash received, net of taxes paid or refunded, and increased by cash invested.

The last part of the table determines the tax due (or refundable) on each valuation date. The difference between the pre-tax value and the after-tax value represents the deferred tax liability. This deferred tax liability is the amount that would be due (or refundable) if the entire portfolio were sold on that date. If a fraction of the portfolio is divested on a particular date, then a corresponding fraction of the deferred tax amount is actually due or refundable.

With these features of the table in mind, we can see the story it has to tell. Initially, $1,000 is invested, which increases in value to $1,500 on the first valuation date. This 50-percent increase in pre-tax value translates into a 30.15-percent increase in after-tax value,[151] which brings the after-tax value up from $1,000 to $1,302. On that date, an additional $500 is invested, which increases both the pre-tax value and the after-tax value by this amount.

[151] $1.3015 = 1.50^{0.65}$.

The second valuation date is triggered by the receipt of $100, which could be the proceeds of a sale of a security, or it might be the receipt of an interest or dividend payment. The label is irrelevant: under this scheme, all receipts are regarded as partial divestitures of the underlying portfolio. By this point, the pre-tax value has grown by an additional 35 percent, which is equivalent to a 21.54-percent growth in after-tax value. The after-tax value grows from $1,802 to $2,190. The difference between this after-tax value of $2,190 and the pre-tax value of $2,700 is $510 of deferred tax liability. The receipt of $100 represents a 3.70-percent divestiture, so the tax due is 3.70 percent of $510, or $19. The $81 left over after paying this tax is reinvested.

In the third accrual period, the portfolio suffers a decline in value: the pre-tax value drops to $2,700, and the after-tax value drops to $1,982. At this point, the investor withdraws $300 pre-tax, but reinvests nothing. Although the portfolio has declined in value during the accrual period, because of prior appreciation the pre-tax value is still higher than the after-tax value. Accordingly, a tax of $41 is due, and the investor is left with $259 for consumption.

In the final accrual period, a further decline in value causes the pre-tax value of $1,000 to dip slightly below the after-tax value of $1,098. As a result, there is now a deferred tax asset of $98 rather than a deferred tax liability. The investor does a wash sale on 50% of the portfolio, thereby turning half of this deferred tax asset into cash that can be reinvested. Although this wash sale costs the government revenue during the current period, the investor reaps no advantage: the pre-tax value of the portfolio goes up but the after-tax value remains unchanged.

The rest of this section gives the formulas for calculating the tax on a portfolio basis. The formulas are given "recursively": the values of each item on a particular valuation date are determined solely on the basis of amounts determined on that date or on the immediately

preceding valuation date. The following notation is used for amounts determined as of the nth valuation date:

V_n = pre-tax value (before cash flow adjustments);
V'_n = pre-tax value (after cash flow adjustments);
W_n = after-tax value (before cash flow adjustments);
W'_n = after-tax value (after cash flow adjustments);
S_n = cash received;
P_n = cash invested; and
T_n = tax (if positive), or refund (if negative).

The cash flow adjustments on each valuation date are as follows:

$$V'_n = V_n - S_n + P_n;$$
$$W'_n = W_n - (S_n - T_n) + P_n. \tag{13}$$

The pre-tax value is decreased by pre-tax receipts, and the after-tax value is decreased by after-tax receipts. Both are increased by amounts invested (which are always after-tax). If the full after-tax receipts are reinvested, the after-tax amount remains unchanged, which is why this method of computing tax is unaffected by the timing of trading decisions.

The after-tax value on each date is based on the after-tax value on the preceding date adjusted by the portfolio's after-tax yield ratio, based on the growth in pre-tax value:

$$W_n = W'_{n-1}\left(\frac{V_n}{V'_{n-1}}\right)^{1-t} \tag{14}$$

The tax is the proportion of the overall deferred tax liability ($V_n - W_n$) equal to the portion of the value V_n that is implicitly divested by receipt of the proceeds S_n:

$$T_n = (V_n - W_n)\left(\frac{S_n}{V_n}\right) \tag{15}$$

3. *Valuation Incentives*

Part of the beauty of the yield-based method, as applied to a single investment with a single payoff at maturity, was the irrelevance of value at intermediate points during the holding period. We saw, however, that for an investment with a payoff on more than one date, the value on each payoff date was relevant, and therefore a valuation of the investment was necessary on each date that a payment was received.

The expected administrative burden of having to value the investment on each payment date is eased by the incentive that taxpayers would have to value the investment accurately, in order to maximize their after-tax proceeds. Moreover, the precise amount of these proceeds are relatively insensitive to variations in the interim valuation. These consequences follow from the assumption that any tax paid would reduce amounts available to be invested at the same rate as the underlying investment.

Similar logic applies under the portfolio method, because this method treats the entire portfolio as a single investment. The underlying assumption, however, is that the payment of taxes reduces the amount available to be reinvested at the portfolio return, rather than at the return on any particular investment. This is a more realistic assumption, particularly if there were no longer a tax cost to portfolio rebalancing.

The portfolio approach must deal with multiple inflows as well as outflows. The value of the portfolio is relevant on each date that cash is invested, because the new cash only participates in appreciation and depreciation after that date. With the government seen as an equity partner, the infusion of fresh cash is a proper occasion for a "revaluation" of the capital accounts of the investor and the government in the portfolio, where the investor's capital account is represented by the

after-tax value, and the government's capital account is represented by the deferred tax liability.[152]

A valuation triggered by a cash receipt results in a tax liability which, if too low, causes the investor to borrow from the government at a cost that is greater than the portfolio yield; and if too high, causes the investor to lend from the government at a return that is less than the portfolio yield. These consequences are why the investor has an incentive to use an accurate valuation.[153]

No tax is paid when a valuation is triggered by a cash investment. There is therefore no self-correcting mechanism to encourage the investor to use an accurate valuation. This problem can be illustrated by a simple portfolio with two investment dates and a single payoff on liquidation. Consider, for example, a portfolio with an initial investment of $50, a further investment of $50 at a later date, and a final payoff of $150. The tax imposed on the final payoff date will depend on the valuation on the date of the second investment, as shown in Table 7.

Here, the investor incurs the lowest tax liability if the interim value is $50; that is, if there is no gain or loss during the first accrual period. There is no relationship, however, between the optimal value to the investor, and the portfolio's actual value on that date. Thus, there can be no assurance in general that the investor will be motivated to seek an accurate valuation.

The table also shows how little the investor will care. The range of possible assumed values encompasses a doubling, from $40 to $80, yet the tax varies by less than $1. The yield-based method is able to correct for tax deferral without being too much affected by interim valuations.

152 *See* Treas. Reg. § 1.704-1(b)(2)(iv)(f).

153 *See supra* Part III.G.2 (p. 337).

TABLE 7

Assumed Interim Value	$40.00	$50.00	$60.00	$70.00	$80.00
Amount Invested	50.00	50.00	50.00	50.00	50.00
New Interim Value	$90.00	$100.00	$110.00	$120.00	$130.00
First Accrual Period					
Pre-Tax Yield Ratio	0.8000	0.0000	1.2000	1.4000	1.6000
After-Tax Yield Ratio	0.8750	0.0000	1.1258	1.2445	1.3573
After-Tax Value	$42.25	$50.00	$56.29	$62.22	$67.87
Amount Invested	50.00	50.00	50.00	50.00	50.00
New After-Tax Value	$93.25	$100.00	$106.29	$112.22	$117.87
Second Accrual Period					
Pre-Tax Yield Ratio	1.6667	1.5000	1.3636	1.2500	1.1530
After-Tax Yield Ratio	1.3938	1.3015	1.2234	1.1561	1.975
After-Tax Value	$129.96	$130.15	$130.03	$129.74	$129.35
Tax Liability	20.03	19.85	19.97	20.26	20.66

This insensitivity to valuation is more than an administrative convenience. As we saw earlier, the concept of valuation itself is theoretically problematic.[154] Ideally, valuations would be unnecessary; but if valuations must happen, it is best if there is no pressing need for precision.

4. *Indefinite Rollover*

The examples and the formulas set forth above treated a cash receipt from an investment in the portfolio as an occasion for taxing a slice of the portfolio's deferred tax amount, even if the proceeds were reinvested. Yet if the portfolio was truly regarded as a single invest-

[154] *See* Part II.A.2 (p. 289).

ment, changes in the composition of the portfolio would be disregarded. Only new cash being invested, or investment proceeds being withdrawn, would be occasions for valuation and possible imposition of tax.

This version of the portfolio approach would allow a tax-free rollover for all investment trading. Only when proceeds were withdrawn for consumption would a tax be imposed. Yet the end result would be nothing like a consumption tax. A consumption tax makes no systematic attempt to tax investment returns, although these returns indirectly show up in the tax base when they affect consumption. The yield-based method, by contrast, taxes investment returns relentlessly. No matter how long the tax is put off, the government gets its due.

Not all income is consumed; some is passed to the next generation. Three choices are possible upon the death of the taxpayer. The first is to forgive the deferred tax liability. This approach resembles the exemption of capital gains at death.[155] This forgiveness, if coupled with indefinite rollovers, would severely undercut the claim of the yield-based method to be an income tax. Moreover, valuation misstatements would no longer have relatively benign consequences. Instead, taxpayers would be strongly disposed to under-value, in the hopes that the deferred tax will never come due.

The second choice on death is to apportion the deferred tax liability among the legatees, who must pay the tax when they withdraw the funds for consumption. This approach is analogous to the ill-fated attempts to apply carryover basis at death.[156] While this choice does not in principle impair the tax base, the government may have to wait a very long time to cash in its share of the gains. The use of the yield-based method, rather than an accretion system, is an attempt to ac-

[155] I.R.C. § 1014(a).

[156] *See* Pub. L. No. 94-455, § 2005(a)(2), 94 Stat. 1872 (1976) (enacting I.R.C. § 1023); Pub. L. 96-223, § 401(a), 94 Stat. 229, 299 (1980) (repealing I.R.C. § 1023).

commodate concerns about liquidity. Yet the effect of indefinite rollover coupled with carryover of deferred tax liability is to defer tax collection far beyond the point that liquidity needs would normally require.

The final choice is to treat death as a final act of "consumption" by transmitting one's wealth to others. A deemed portfolio liquidation would require investments to be marked to market, and the entire deferred tax liability would become immediately payable to the government. This choice is analogous to taxing capital gains at death. The same liquidity concerns arise here that arise under the estate tax today. Indeed, taxation at death under the yield-based method could be an effective substitute for the estate tax.

V. LIFE UNDER THE YIELD-BASED TAX

Is taxation under the yield-based method a crazy dream, or could it really happen? The dynamics are so unfamiliar to our current way of thinking that it takes a real effort of imagination to consider what such a scheme would be like in real life.

A. *Extensions*

A practical implementation of the yield-based method would need to address deferrals that arise in transactions that, taken alone, do not involve an investment of funds. The absence of such an investment makes it difficult to speak of "yield" on invested capital. The portfolio approach, however, provides a context in which these transactions can be properly taken into account.

1. *Liabilities*

The realization requirement works badly with liabilities, because the realization event happens when the liability is extinguished but the cash appears when the liability is incurred. If debt is discharged for less than its face amount, the debtor has taxable income at that time,[157] but no cash with which to pay tax. The resulting liquidity problem is the central challenge of bankruptcy tax planning. The tax law provides some relief for insolvent or bankrupt debtors by allowing this taxable income to be offset by tax benefit carryforwards and depreciable asset basis,[158] but other debtors face an immediate tax liability, even though the cash associated with the income may have been spent much earlier, when the liability was incurred.

157 I.R.C. §§ 61(a)(12), 108(e)(1).

158 I.R.C. § 108(a)(1)(A), (B), (b).

Liabilities have a yield, but this yield represents a cost to the borrower rather than an investment return. The yield-based formulas could be applied to this yield, so that the borrower receives a tax benefit that scales back the yield from its pre-tax level to an after-tax level that equals the pre-tax yield times one minus the tax rate. The retrospective feature of the yield-based method makes it easy to apply to liabilities such as written options, in which the obligor receives a fixed amount of cash in return for an obligation to pay a contingent amount in the future.

Applying the yield-based method to each separate liability raises the same problems of non-linearity that are raised applying this method to assets separately. Moreover, retrospective taxation of liabilities does nothing to address the liquidity problem that arises because the cash is received up front.

The answer is to throw liabilities into the portfolio. Liabilities are a way of potentially enhancing the yield on the investor's equity investment. The yield-based method can be applied to a portfolio with liabilities by measuring changes in the value of the investor's net equity interest in the portfolio from one valuation date to the next. The non-linearity problem disappears because the yield on the liability affects the yield-based tax only as it affects the overall yield on the equity investment. The liquidity problem can be addressed by treating the *drawdown* of the liability, but not its repayment, as a realization event. This means that the discharge of a liability for less than its face amount will not trigger a tax liability at that time, but the yield-based method will ensure that the investor reaps no economic advantage from the tax deferral. On the other hand, borrowing against an appreciated asset portfolio can be quite properly made into a realization event, enabling the government to collect its tax while there is cash on the table. Again, the resulting acceleration of tax does no harm to the investor.

This focus on the yield on the portfolio's net equity neatly sidesteps the question whether the investor is using deferred taxes to

reduce borrowing or to increase investments. The investor seeks the best return on his or her equity, and the decision to borrow is a choice affecting risk and expected return, just like the choice of investments. The value of tax deferral to the investor is the enhanced return that can be earned on this equity, regardless of whether the enhancement arises from avoided borrowing costs or additional investment income.

2. *Financial Contracts*

Forward contracts and swaps generally involve no investment or borrowing, although they can be designed with up-front payments that are in effect embedded loans.[159] For example, in a typical interest rate swap, one party might agree to make a series of payments equal to a floating rate of interest on an agreed-upon principal amount, which the other party agrees to make a series of fixed payments, representing a fixed rate of interest on that principal amount.

In cases not involving an embedded loan, it is meaningless to refer to a yield on the contract, because there is no amount invested by either party. The periodic net payments do not create a net investment by the party that is making the payments, because these payments represent incurred losses rather than an investment in an asset. A position in a swap or forward contract can acquire significant value prior to its maturity, but the cost basis is generally zero, so that its "yield" is potentially infinite. It is difficult to imagine applying the yield-based method to such an instrument, yet these instruments create a tax deferral opportunity if they are not taxed on an accretion basis.[160]

As with liabilities, the way to apply the yield-based method is to take these instruments into account when valuing the portfolio. These

[159] In cases where there is an embedded loan, the regulations may require that it be separately taxed as such. Treas. Reg. § 1.446-3(g)(4).

[160] *See* Reed Shuldiner, *A General Approach to the Taxation of Financial Instruments,* 71 Tex. L. Rev. 243, 308 (1992).

contracts require some creditworthiness on the part of the obligor, which means that as a practical matter there is always an asset portfolio, and the investor's goal in entering into these contracts is to improve the portfolio yield. The connection is blatant in the case of covered written call options, where the obligor owns the asset subject to the call, but more generally there is a pool of assets that stands behind the financial contract obligation.[161]

3. *Investments Issued for Services*

A vexing problem is created by the acquisition of an investment in exchange for services. Where the investment can be readily valued, this value can be taken into account as taxable income at the time the services are provided. The difficult case is the sole shareholder who works for a low salary while building up the corporation. Great fortunes have been amassed in this manner, perhaps abetted by the fact that in cases like these the realization requirement effectively defers tax on compensation income as well as the return on capital.

In theory the yield-based method can eliminate this tax deferral. There is a risk, however, that it might go too far. Suppose a business is built up to a value of $1 million with an investment of $100,000 in cash plus $100,000 worth of "sweat equity" in the form of the investor's own efforts (which, for simplicity, will be assumed to have been provided at the same time as the cash). If only the cash investment is taken into account in computing yield, then upon a sale of the business for $1 million, a tax of $553,316 will be due.[162] This tax is too high because the after-tax value of the services ($65,000 in this case)

[161] Even in the case of "pure" gamble, there is an investment component; it takes money to place a bet.

[162] Using Equation (9) and assuming a tax rate of 35%: Tax = $1,000,000 (1 – ($1,000,000/$100,000)$^{-0.35}$) = $553,316.

should be taken into account as part of the initial investment, which reduces the tax to $467,745.[163]

In the case of a privately held corporation, the value of the owner-manager's services can be difficult to value. Unlike the periodic valuations that are required under the portfolio approach to reflect cash inflows and outflows, the amount of tax is highly sensitive to the valuation, and taxpayers will have an incentive to fight the issue with the IRS. Worse still, these services are often provided continuously over an extended period of time, over which time the value of the business may be changing as well.

The only practical solution is to rely on general canons of reasonable compensation,[164] and to value the business continuously by interpolation from values at stated intervals. This achieves only rough justice, but does not appear to create opportunities to magnify any resulting distortions through tax-motivated investment strategies. Even a crude approximation to the theoretically correct result is a vast improvement over the present system, which allows enormous tax deferral benefits to those who think of themselves as the "self-made" rich.

4. *Changing Tax Rates*

All of the formulas and examples in this article assume a constant tax rate. History teaches otherwise; Congress is always tinkering with the tax rates.[165] Mechanically, the necessary adjustment under the yield-based method is straightforward. Each date on which the tax rate changes is a valuation event, regardless of whether any cash flows in

163 Tax = $1,000,000 (1 – ($1,000,000/$165,000)$^{-0.35}$) = $467,745.

164 *See, e.g.*, I.R.C. § 162(a)(1).

165 *E.g.*, Omnibus Budget Reconciliation Act of 1993, Pub. L. No. 103-66, § 1302(a)(1), 107 Stat. 312, 461 (raising top marginal rate to 39.6%); Tax Reform Act of 1986, Pub. L. No. 99-514, § 101, 100 Stat. 2085, 2096–99 (setting top marginal rate at 28%).

or out of the portfolio at that time. The portfolio is valued on a pre-tax basis, and the after-tax value is determined based on the rates previously in effect. On the next valuation date, the after-tax value is determined by applying the new tax rate to the pre-tax yield ratio. This procedure ensures that changes in value before and after the change in rates are each taken into account at the appropriate rate.

Unfortunately, changes in tax rates raise the stakes in the valuation game. Taxpayers will seek to manipulate the valuation to shift income into the period with the lower tax rate: if rates go down, taxpayers will seek a low valuation; if rates go up, they will seek a high valuation. All of this is contrary to the spirit of the yield-based approach, which is to make accuracy in valuation a relatively unimportant matter.

Ideally, Congress would minimize the frequency and magnitude of rate changes. To expect such self-restraint is unrealistic, since revenue needs change with the fortunes of war and peace, and with the business cycle. Yet indeterminacy in valuation creates less opportunity for tax avoidance with changing rates than under current law. The realization requirement makes it fairly easy to time realization events to occur in the period of lower tax rates.[166] Indeed, much of the early tax law affecting the timing of income arose in the context of disputes over taxable years during the First World War, when tax rates were significantly higher than in the immediately preceding or succeeding years.[167] It was changing tax rates, not the time value of money, that made the timing of income so critical.

166 *See supra* note 20 and accompanying text. When gain is recognized under the installment method, historically the applicable tax rate has been the rate for the year of payment rather than the year of sale. *See* Picchione v. Comm'r, 440 F.2d 170, 172–73 (1st Cir. 1971); Snell v. Comm'r, 97 F.2d 891, 893 (5th Cir. 1938); Klein v. Comm'r, 42 T.C. 1000, 1004 (1964).

167 In 1917, corporations were subject to an excess profits tax with rates up to 60%. War Revenue Act of 1917, Pub. L. No. 65-50, § 201, 40 Stat. 300, 303.

5. *Discontinuities in Value*

Income that is earned suddenly cannot be reinvested while it is being earned. This was the lesson taught by the examples using discrete compounding, where income is assumed to be earned instantaneously at the end of each compounding period.[168] Yet whether income is earned continuously is a matter of empirical fact, not of accounting convention.

Consider income earned by winning a $100 coin toss. There was no intermediate point at which one had earned $50, which could have been reinvested before the toss was complete. If the tax on this income is paid immediately, the taxpayer has not enjoyed any deferral benefit, and has not made use of the government's money.

A coin toss is an artificial example, but the same effect occurs whenever an investment changes in value instantaneously. Continuous functions have the property that one cannot get from point x to point y without passing through each intermediate point between them.[169] Investment values lack this property even when the investments are publicly traded. For the 1987 market crash, investors with standing "stop loss" orders to sell stocks that fell below a fixed price discovered that there were no takers at that price. Instead, the trading price jumped from a value above this fixed price to a value significantly below, without stopping at any points in between.[170]

Small instantaneous jumps in value can occur frequently as investments are buffeted by random events. So long as these jumps are small in relation to total amounts of income earned over time, the resulting distortions in the application of the yield-based method are small enough to be disregarded. The yield-based method could pro-

168 *See supra* Part III.E (p. 325).

169 G.H. HARDY, A COURSE OF PURE MATHEMATICS 190–91 (10th ed. 1963).

170 Even in more tranquil times, investors have been cautioned that price jumps can subvert a stop loss order. *See, e.g.,* LOUIS ENGEL, HOW TO BUY STOCKS 89 (6th ed. 1976).

vide special relief for a jump in value that is extraordinarily large, by adjusting the after-tax value of the portfolio at that time by an amount equal to the pre-tax change in value multiplied by the tax rate. This adjustment ensures that no account is taken of tax deferral over the (instantaneous) period of the jump.

B. *Evaluating the Yield-Based Tax*

1. *Valuation and Liquidity*

The goal of retrospective taxation is to eliminate the evils of the realization requirement without creating the valuation and liquidity problems inherent in accretion schemes. On the liquidity side, the yield-based approach acquits itself well. Not only is no tax imposed until a realization event, it is possible to be fairly relaxed about allowing tax-free rollover, even to the point of taxing income only when withdrawn for consumption. To achieve this result without compromising the income tax base would be a significant step forward.

Valuation is more of a mixed story. Given the practical and theoretical problems of valuation, the reliance of the yield-based approach on frequent portfolio valuations is a serious drawback, at least in cases where the portfolio includes illiquid assets. Fortunately, the yield-based method does not require precise valuations in order to work passably well. As a result, hard-to-value assets can be valued by means that do not purport to be terribly accurate, such as valuing privately held companies at the book values shown on their financial reports. Values on odd dates can be weighted averages of month-end or year-end figures. In short, while the yield-based method seeks valuations, it is content with valuation proxies.

2. *Fairness*

Part of the motivation for correcting the effects of the realization requirement was to achieve greater fairness. By eliminating the effects of deferral, the yield-based method treats investors that defer realization no more favorably than those that are taxed currently on their income, thereby promoting horizontal equity. The realization requirement tends to favor the rich, so eliminating its benefits favors vertical equity as well.

On its face, the yield-based approach appears to be pro-government. Superficially, there is justice to this charge: eliminating the benefits of deferral will raise the effective tax burden on income from capital. But the yield-based approach has nothing to say about the optimum level of tax revenues generally. If more tax is imposed on income from capital, less tax can be imposed on other things.

Moreover, the yield-based method makes possible other changes to the tax system, some of which are profoundly pro-taxpayer. Perhaps the most significant would be the elimination of the corporate income tax, which is best justified under current law as a crude antidote to the distortions caused by the realization requirement.[171] Also, as noted earlier, the yield-based approach, coupled with a realization at death, could replace the estate tax.[172]

The non-linear nature of yield-based taxation makes it relatively unforgiving of losses. When applied to investments on a separate basis, the yield-based approach generates a smaller tax benefit than would be allowed under current law (assuming that the taxpayer has sufficient gains to absorb the loss).[173] The yield-based approach is

171 *See supra* note 35 and accompanying text.

172 *See supra* Part IV.C.4 (p. 355). Reflecting both long standing ambivalence towards the estate tax as well as current politics, there have been recent partisan appeals to eliminate the estate tax altogether. *See, e.g.,* H.R. 525 and H.R. 902, 105th Cong., 1st Sess. (1997).

173 *See supra* Part IV.A (p. 341).

gentler when applied on a portfolio basis, because losses freely offset gains in the overall valuations. But when the portfolio as a whole declines in value, the yield-based approach treats the portfolio just as harshly as a single investment, since it treats the entire portfolio as a single investment. In the extreme case, a portfolio that declines in value to zero produces no tax benefit at all.[174]

Yet current law is also unkind to those who lose money overall. Capital losses are generally deductible only against capital gains. An investor that generates overall net losses gets no tax benefit. Indeed, here the yield-based approach is less harsh than current law, because investors that suffer less-than-total losses in overall value can get tax refunds. Furthermore, under the yield-based approach the tax law could become more flexible in allowing wash sales to generate these refunds, and might go further by allowing mark-to-market elections whenever the taxpayer chooses.

3. *Behavioral Effects*

The yield-based method would have some salutary effects on investor behavior. The most prominent would be the elimination of the lock-in effect that discourages investors from selling appreciated assets under current law. By eliminating the tax disincentive to sell, the yield-based method allows capital to flow more freely to its most productive uses.

An accretion system of taxation would also eliminate the lock-in effect. But accretion would substitute a different evil: forced sales to pay taxes. A yield-based system approaches the ideal of neutrality, in which investors sell no sooner, and no later, than they would in a world without taxes.

Eliminating the lock-in effect would remove a principal justification for a favorable rate of tax on capital gain.[175] The favorable rate of

[174] *See supra* Part IV.A (p. 341).

[175] *See supra* note 12 and accompanying text.

tax on capital gains has also been defended as an incentive for risky investments.[176] While a favorable rate does make high risk, high return investments more attractive by enhancing the potential return, the yield-based approach promotes risk-taking more directly by making these investments less risky. As noted above,[177] the yield-based approach makes it easier to allow tax refunds for portfolio losses. It compresses the range of possible pre-tax portfolio yields to a narrower range of after-tax yields. As a result, risk-averse investors will have a greater tolerance of risky investments.

4. *Simplicity*

On the face of it, the yield-based approach looks like a step towards more complexity. The formulas are odd and unfamiliar. Portfolio-wide valuations are necessary, possibly at frequent intervals. Yet under the surface are significant steps towards simplicity. One is the distinction between return on capital and recovery of capital. There is no "fruit" or "tree"; all is vegetation.

Speaking of vegetation, large swaths of the tax law could become deadwood. Much of the complexity of current law steps from attempts to control the distortions caused by deferral. Restrictions on loss deductibility, wash sales, and straddles could find their way to the dustbin, joined by the regulations on contingent payment debt. More radical changes, such as the elimination of the corporation income tax or the estate tax, would be simplifications on a colossal scale.

Even so, it would be grossly premature to assert that adopting the yield-based approach would be a simplifying move overall. The yield-based approach has been presented here in a schematic way under laboratory conditions. Current law has to contend with the messiness of the real world. The yield-based method has been presented here

[176] As an incentive for risky investments, however, the capital gains preference is badly flawed. *See* Cunningham & Schenk, *supra* note 12, at 340–41.

[177] *See supra* Part V.B.2 (p. 366).

without any mention of progressive rates, taxation of family units, income (such as trust income) with contingent ownership, tax incentives for savings and investment, sourcing of income for purposes of international taxation, and timing of tax collections. All of these issues, and more, would need to be addressed before a concrete proposal could be devised using the yield-based method.

C. *Could It Really Happen?*

The obstacles to implementing a yield-based tax are severe. Most of us have learned to cope with the problems of current law, and prefer known to unknown evils. Moreover, the dynamics of the yield-based tax can seem odd at first glance. For example, the sale of an investment at a loss can produce a positive tax liability if the value of the overall portfolio has gone up. Upon reflection, however, such a result can be seen as perfectly normal; after all, portfolio performance better measures gains in wealth and ability to pay.

Translating a concept into legislation requires attention to detail on a scale far beyond the scope of this paper. It also requires political support. Correcting for the effects of the realization requirement is a favorite concern of the tax law academic community, but it lacks a political constituency. Indeed, at present the political wind is blowing the other way, as politicians in both parties are weighing proposals to further reduce the effective tax burden on income from capital.[178] Even the so-called radical reform proposals coming from Washington mainly reduce rate progressivity without addressing the realization requirement.[179] And a shift to a consumption tax would throw in the towel completely.

[178] *See* supra the discussion in note 10 regarding recent lowering of capital gains rates.

[179] *See, e.e.,* CONG. REC. E1461 (daily ed. July 19, 1995) (statement of Rep. Armey) (describing proposal to replace existing Code with a flat tax on all income); 141 CONG. REC. S3416 (daily ed. Mar. 2, 1995) (statement of Sen. Spector) (describing proposal to impose a flat tax on earned income).

Even if it never happens, the yield-based tax is worth studying. Deferral remains a concern in particular areas of the tax law, such as offshore investments and long-term debt and leases. The effectiveness of any particular proposals to address the realization requirement is best judged against the standard that the yield-based method can provide. Deviations from the ideal might be defended on the grounds of feasibility or practicality, but at least they would be done with an understanding of what these deviations are. Finally, the yield-based method offers insights into the dynamics of the time value of money. Many of these insights are surprising and counter-intuitive. Even if the yield-based method does not usher in a new tax system, it sheds light on the current system.

VI. APPENDIX

A. *Immunity to Strategic Trading*

This section offers a proof that the yield-based method is immune to strategic trading.

The proposition to be proved here is that if an investment is purchased at price P, grows (or shrinks) to S_{1p} in market value at some later time, and then further grows (or shrinks) to a final selling price of S_{2p}, then the tax that would be imposed on the final sale leaves the taxpayer in exactly the same position as if the taxpayer had sold the asset at the earlier time for S_{1p}, and paid tax (or got a refund on a claimed loss). The taxpayer in that circumstance is assumed to have reinvested the after-tax proceeds in the same type of asset, earning whatever the pre-tax yield was on that asset during the remainder of the holding period.

Under the first scenario, in which no tax is paid until the final disposition, the after-tax amount is:

$$S_{2a} = PR_{2a} = PR_{2p}^{1-t.} \tag{16}$$

Under the second scenario, a tax is imposed (or a tax refund is made), when the taxpayer sells the asset for S_{1p}:

$$T = PR_{1p}(1 - R_{1p}^{-t}). \tag{17}$$

The amount left to reinvest, after paying this tax, is:

$$S_{1a} = PR_{1p}^{(1-t)}. \tag{18}$$

Let $R_{2'p}$ be the pre-tax yield ratio over the remaining portion of the holding period:

$$R_{2'p} = R_{2p}/R_{1p}. \tag{19}$$

The after-tax yield ratio is defined in terms of the pre-tax yield ratio as in equation (7):

(20) $$R_{2'a} = R_{2'b}^{(1-t)}.$$

The final after-tax proceeds is the reinvested amount, given by equation (18), times the after-tax yield ratio, given by equation (20):

(21) $$S_{2a} = S_{1a}R_{2'a} = PR_{1b}^{(1-t)}R_{2'b}^{(1-t)}.$$

Substituting into equation (21) the value of $R_{2'b}$ given in equation (19) gives:

(22) $$S_{2a} = PR_{1b}^{(1-t)}[R_{2b}/R_{1b}]^{(1-t)} = PR_{2b}^{1-t},$$

which is the amount obtained for S_{2a} in equation (16) under the first scenario.

Although the foregoing deals with only a single wash sale, the same results follow if any number of wash sales are performed during the holding period. The general case can be established by mathematical induction.[180] First, its truth in the case of n wash sales is assumed. If an additional wash sale is made, one can apply the case of a single wash sale to the holding period between the purchase and sale immediately before and after the additional wash sale. Thus, the taxpayer has the same amount of after-tax proceeds upon the sale immediately following the additional wash sale as would have been realized had the additional wash sale not been made. The investment results for the balance of the holding period are unaffected, so the equivalence of final after-tax proceeds is unaffected by the additional wash sale. ∴

B. *Incentive for Accurate Valuation*

This section proves in general that an investor can maximize the after-tax proceeds from an investment with two payments by calculating the tax on the interim payment using an assumed value for the investment that is equal to its actual value.

[180] Under mathematical induction, a statement about numbers is proven for all numbers if (i) it is proven for the first number 1, and (ii) assuming the statement is true for n, it is proven for n+1.

The notation is as follows:

P = purchase price
P_2 = portion of purchase price allocated to the final payment
S_1 = interim payment
S_2 = final payment
V_1 = *assumed* value, just before the interim payment
V_1^* = *actual* value, just before the interim payment
T_1 = tax on the interim payment
t = tax rate
A_2 = final after-tax proceeds

The final after-tax proceeds consist of two parts. First, there is the final payment, less the tax paid on the final payment. Second, there is the reinvestment of the interim payment, less the tax paid on the interim payment; the net after-tax interim payment is assumed to be reinvested at the same yield as the investment itself.

To simplify the formulas, it will be useful to refer to P_2, which is the portion of the original investment that is not considered to have been implicitly divested by receipt of the interim payment. Since what is left after receipt of the interim payment is assumed to be worth $V_1 - S_1$, the value for P_2 is given by:

$$P_2 = P\left(\frac{V_1 - S_1}{V_1}\right). \tag{23}$$

The tax on the interim payment is computed using equation (9) by treating the interim proceeds as the proceeds of a single-payment investment with a payoff of S_1 and a cost of P minus P_2:

$$T_1 = S_1\left(1 - \left(\frac{S_1}{P - P_2}\right)^{-t}\right) = S_1 - S_1\left(\frac{S_1}{P - P_2}\right)^{-t}. \tag{24}$$

The after-tax interim proceeds of S_1 minus T_1 are assumed to be reinvested at the *actual* yield on the investment over the balance of the holding period. Over this interval, the investment grows from V_1^* minus S_1 to the final payment of S_2. The ratio of these amounts is the

pre-tax yield ratio to be used in calculating the after-tax proceeds of reinvesting the interim payment, based on equation (7):

$$(S_1 - T_1)\left(\frac{S_2}{V_1^* - S_1}\right)^{1-t}.$$

Substituting for T_1 in this expression using equation (24) yields:

$$S_1\left(\frac{S_1}{P - P_2}\right)^{-t}\left(\frac{S_2}{V_1^* - S_1}\right)^{-t}.$$

The other component of the after-tax proceeds is the amount S_2 received at maturity, less any tax payable at that time on this payment. This final payment is treated as the payoff on a single-payment investment with a cost of P_2 allocated to it. The pre-tax yield ratio is S_2/P_2, so the after-tax yield ratio is $(S_2/P_2)^{1-t}$, and the after-tax proceeds are:

$$P_2\left(\frac{S_2}{P_2}\right)^{1-t}.$$

Putting these last two expressions together gives the aggregate after-tax proceeds A_2:

$$A_2 = P_2\left(\frac{S_2}{P_2}\right)^{1-t} + S_1\left(\frac{S_1}{P - P_2}\right)^{-t}\left(\frac{S_2}{V_1^* - S_1}\right)^{1-t}. \tag{25}$$

Equation (25) shows the after-tax proceeds A_2 as a function of P_2, which in turn is a function of the assumed value V_1, as given by equation (23). If A_2 were plotted on graph paper as a function of V_1, A_2 would be seen to rise and then fall as V_1 increases, and the slope of the curve would be level at the point where it attains a maximum value. In terms of the calculus, because A_2 is a continuous and continuously differentiable function of V_1, it attains a local maxi-

mum[181] at the point where its partial derivative $\partial A_2/\partial V_1$ is equal to zero.

The point where A_2 has a zero derivative can be found using the chain rule for derivatives:

$$\frac{\partial A_2}{\partial V_1} = \frac{\partial A_2}{\partial P_2} \cdot \frac{\partial P_2}{\partial V_1}. \tag{26}$$

The second factor, $\partial P_2/\partial V_1$, can be evaluated based on the formula for P_2, given by equation (23):

$$P_2 = P\left(\frac{V_1 - S_1}{V_1}\right) = P\left(1 - \frac{S_1}{V_1}\right) = P - \frac{PS_1}{V_1}. \tag{27}$$

The derivative is:

$$\frac{\partial P_2}{\partial V_1} = \frac{PS_1}{V_1^{-2}}, \tag{28}$$

which is always a negative number, so the only way that $\partial A_2/\partial V_1$ can be equal to zero is for its first factor $\partial A_2/\partial P_2$ to be zero.

The formula for $\partial A_2/\partial P_2$ can be found by differentiating the formula for A_2 in equation (25) with respect to P_2:

$$\frac{\partial A_2}{\partial P_2} = tS_2^{1-t}P_2^{t-1} - tS_1^{1-t}\left(\frac{S_2}{V_1^* - S_1}\right)^{1-t}\left(P - P_2\right)^{t-1}. \tag{29}$$

The derivative equals zero when:

$$tS_2^{1-t}P_2^{t-1} = tS_1^{1-t}\left(\frac{S_2}{V_1^* - S_1}\right)^{1-t}\left(P - P_2\right)^{t-1}. \tag{30}$$

In equation (30) both t and S_2^{1-t} cancel out:

$$P_2^{t-1} = S_1^{1-t}\left(\frac{1}{V_1^* - S_1}\right)^{1-t}\left(P - P_2\right)^{t-1} \tag{31}$$

The right-hand side can be regrouped:

[181] Or minimum, since smooth curves are flat at the valleys as well as the peaks. Here, the point at which $\partial A_2/\partial V_1 = 0$ can be shown to be a peak rather than a valley by looking at the *second* derivative $\partial^2 A_2/\partial V_1^2$, which shows the slope of A_2 to be decreasing at that point. The calculations are omitted here.

(32) $$P_2^{t-1} = \left(\frac{\left(V_1^* - S_1\right)\left(P - P_2\right)}{S_1} \right)^{t-1},$$

and the exponents dropped:

(33) $$P_2 = \frac{\left(V_1^* - S_1\right)\left(P - P_2\right)}{S_1}.$$

From equation (27) it can be seen that

(34) $$P - P_2 = \frac{PS_1}{V_1}.$$

Substituting the right-hand side of equation (34) for $(P - P_2)$ in equation (33), and substituting the right-hand side of equation (23) for P_2 on the left-hand side of equation (33) yields:

(35) $$P\left(\frac{V_1 - S_1}{V_1} \right) = \left(\frac{V_1^* - S_1}{S_1} \right) \cdot \frac{PS_1}{V_1}.$$

In equation (35), P drops out of the numerator on both sides, V_1 drops out of the denominator on both sides, and S_1 cancels out on the right-hand side:

(36) $$V_1 - S_1 = V_1^* - S_1$$

which is true only if

(37) $$V_1 = V_1^*$$

Thus, the derivative $\partial A_2 / \partial V_1$ equals zero, and the maximum value of A_2 is attained, where V_1 is equal to V_1*; that is, where the assumed value of the investment at the time of the interim payment is equal to its actual value. ∴